1 Introduction

The project **"A Longitudinal Study of Mathematical Reasoning: Student Development and School Influences"** aims to investigate how students develop competencies in mathematical reasoning over time and how schools and teachers promote this development. This project has been funded by ESRC for four years to conduct a large-scale longitudinal study in which above-average students from about 70 schools are tested annually for three years on their understanding of different aspects of mathematical reasoning. The first survey was administered in June 2000 to around 3000 students in top sets or bands in mathematics who were nearing the end of Year 8. The same students have been tested again using new or modified questions, in the summer term 2001. This will be repeated again in 2002. In addition to the student data, all the teachers of the classes involved in the survey will complete a teacher questionnaire to provide relevant personal data and a school questionnaire to provide information on the location, intake, status and procedures of the school. The data from the tests will be analysed to identify school, curriculum, teaching and student factors associated with competence and progress in mathematical reasoning.

In this report, we model the total scores for year 9 constructive proof in geometry and algebra (27 in algebra and 15 in geometry). We use three important variables: year 8 proof score, year 8 baseline test score and key stage 3 6-8 test score as the baseline information in the data analysis.

The report is organised as follows: Section 2 describes the methodology of data analysis and statistical strategies for dealing various statistical issues occurring during the data management and analysis. Section 3 models and predicts the year 9 proof score based on the year 8 proof score and other important variables. Similar analyses are performed on the year 9 proof score based on year 8 baseline score and 6-8 key stage score in Section 4 and 5, respectively. The brief summary is given in the last section.

2 Data and Methodology

2.1 Data description

In this analysis, we model the total raw scores for Algebra (A1, A2, A4, L1) and Geometry (G1, G2, G4) as a bivariate response. The main purpose of this kind of modelling is to obtain estimates of the correlations between school effects on the two outcomes, if any, and also to study correlations at student level. A subsidiary purpose is to obtain more precise estimates of all effects, both fixed and random.

Data were available on 1985 students in 90 classes spread over 59 schools. Not all classes had teacher data, and not all the student data were complete.

The following table displays the sample sizes according to the valid values of three baseline variables, Year 8 proof score, Year 8 baseline test score, and Key Stage 3 test score

Proof Score	Year 8 proof score	Year 8 baseline test score	Key Stage 3 test score
Geometry	1955	1880	1872
Algebra	1954	1879	1871
Total	3909	3759	3743

The following data were considered for inclusion:

Descriptions of variables in the data analysis

Variables	Description
Baseline score	Year 8 proof score Year 8 baseline test score Key Stage 3 test score
school-level:	
administration	1=county, 2=voluntary (VA/VC), 3=or other)
age range 11-16 or 11-18	1=11-16, 2=11-18
% A*-C at GCSE	percentage
gender	1=girls only, 2=mixed
area (one of three)	1=rurual, 2=urban, 3=suburban
year-9 size	school size
% to be entered at lev6-8 at KS3	percentage
GCSE syllabus	1=EDECEL,2=OCR,3=AQA
maths textbook or scheme in use	1=Key math,2=SMP,3= Vickers, 4= ST(P),5=Other
minutes of maths per week	minutes
existence of maths club	1=Yes,0=No
class-level:	
teacher gender	1=Female, 0=Male
teacher years of experience	year
teacher age	year
teacher degree	1=having a maths degree, 0=otherwise
teacher PGCE or Cert	1=having a PGCE or Cert,0=otherwise
teacher HE	1=having MSc or PhD degress
teacher CPD score	score
teacher membership of a professional assoc.	1=Yes, 0=No
teacher knowledge/use of software	Number of software
teacher total score for CPD, PROF and software	score
student-level:	
gender	1=Girl,0=Boy
age in months	age in months

2.2 Strategy for missing values

There are some missing values for some variables. If a student's response variable (year 9 proof score) is missing, the student is excluded from the analysis. For predictors, if we remove an observation that an independent variable has a missing value, cumulatively the sample size will be drastically reduced. To avoid this problem, statistical imputations have been performed.

For a continuous or numerical variable, we performed imputations in three steps:

Step 1: Check the distribution by looking at its histogram;

Step 2: Fit the probability distribution (such as normal, uniform distribution, etc) and estimate its parameters (such as mean and standard deviation);

Step 3: Impute the missing value by generating a random value from the estimated probability distribution in Step 2.

For a categorical variable, we simply estimate a multinomial distribution from the distribution based on the observed values and the missing value is therefore simulated from the estimated multinomial distribution.

2.3 Variable transformation using ACE algorithm

In multiple regression analysis, there are two types of variables: quantitative (continuous or numerical) and qualitative (binary or categorical).

For a binary variable, we use a simple indicator or dummy variable to represent its effect in a regression model. For example, gender of student is coded as a dummy variable indicating whether a student is a girl or boy.

For a particular categorical variable, a common approach is to create a set of dummy variables. It may be possible to aggregate some groups, but in the absence of well-grounded theory this is often done on an ad hoc basis, since we don't know the most efficient way of doing so. The number of possible ways of collapsing a variable with *n* categories is approximately 2^n-1. For a numeric variable, we usually have no prior knowledge or consensus on whether it should be entered into the regression model in a simple linear form, or in more complex forms such as polynomials, or non-linear functions that cannot be estimated directly. Continuous variables are often cut into groups and treated as if they were categorical. However, the number of possible ways of doing so is very large, intensifying the problem of determining the optimal number of groups and identifying the most relevant class limits.

We search for efficient transformations using the ACE (Alternating Conditional Expectation) algorithm (Breiman and Friedman, 1985; De Veaux, 1989), a technique for finding non-parametric transformations of both the independent and dependent variables in multiple linear and generalised linear regression models such that the relationship between them is as linear as possible. The ACE results indicate if transformations are necessary and, if so, to suggest empirical parametric or non-parametric transformations of the data. Further information is given in Wang and Murphy (1998).

The ACE transformation outputs can be used to study the effects of predictors on the response variable. This can be done by referring to the analytic ACE transformation plots. In general, it is easier to interpret the ACE model from the transformation plot because the plots are easier to conceptualise compared to the mathematical expressions for transformations.

In using transformation plots, a useful observation is that when the transformation for the dependent variable is an increasing function, then the transformed independent variables are positively correlated with the original dependent variable. This is true because higher transformed scores for an independent variable are associated with higher transformed scores of the dependent variable, which in turn maps onto a higher observed score for the dependent variable because its transformation is an increasing function.

2.4 Multivariate multilevel model

Due to the structure of proof data, multivariate multilevel model was employed for the data analysis. We allow three levels of variation, with students at level 2, classes at level 3, and schools at level 4. Level 1 is used to distinguish between the responses in Algebra and Geometry. This is a standard way to set up a multivariate multilevel model (see, for example, Goldstein, 1995, Chapter 4).

2.5 Selection of most parsimonious model using forward and backward approach

It is not wise to include all the independent variables in the statistical analysis for the two major reasons: (1) it will take a huge amount of computing time for multilevel analysis and (2) if the sample size is small there will be a considerable extent of overfitting in the multivariate analysis: many parameters have large standard errors.

We therefore a forward and backward model selection strategy to select the most parsimonious statistical model. This is done by two steps:

Step 1: Select the most parsimonious model using the ordinary multivariate regression method.

Step 2: Multivariate multilevel model analysis of the model selected in Step 1 and backward delete the insignificant variables.

In terms of random effect, insignificant random effects will be removed from models.

2.6 Construction of 5 multilevel models to assess the changes in the variance at different levels

To assess the change in the variance at different levels by adding more predictor variables and changing covariance structures, we built 5 multilevel models:

Model 0: the only fixed effect included, apart from the intercept (‘geo_cons’, ‘alg_cons’), is gender (‘geo_girl’, ‘alg_girl’). The reason for including this very basic model is to check whether there is a gender effect, and whether there is detectable variation in the outcomes at class level, in a model unadjusted for baseline score.

Model 1: the fixed-part predictors are gender and baseline score

Model 2: the fixed-part predictors are gender and baseline score plus other significant variables chosen from forward and backward selection strategy. This means that we are assessing the effects of the other variables, together with any random effects at student, class, and school level, on the proof scores of children with apparently equal attainment in other areas of maths. Thus, for the purpose of school comparison, we are treating the baseline score as an intake measure.

Model 3: as Model 1, but with no random effect of gender at school level.

Model 4: as Model 2, but with no random effect of gender at school level.

In Models 1 and 2, we allow the student-gender effect to vary at both school and student level. This is equivalent to allowing schools to be differentially 'effective' for girls and boys, and also allows that individually boys may vary differently from girls about their predicted scores. In Models 3 and 4 the variance at school level is pooled between girls and boys, in other words, school 'effects' are assumed to be the same for girls as for boys.

2.7 Notations

In the tabulation of the fixed estimates for this and subsequent models:

the prefix 'geo_' indicates an effect on the score in Geometry;

'alg_' indicates an effect on the score in Algebra;

the suffix 'cons' indicates an intercept term;

3 Modeling of the Year 9 Proof Score using the Year 8 Proof Score as Baseline

3.1 Variable Codings as suggested by ACE algorithm

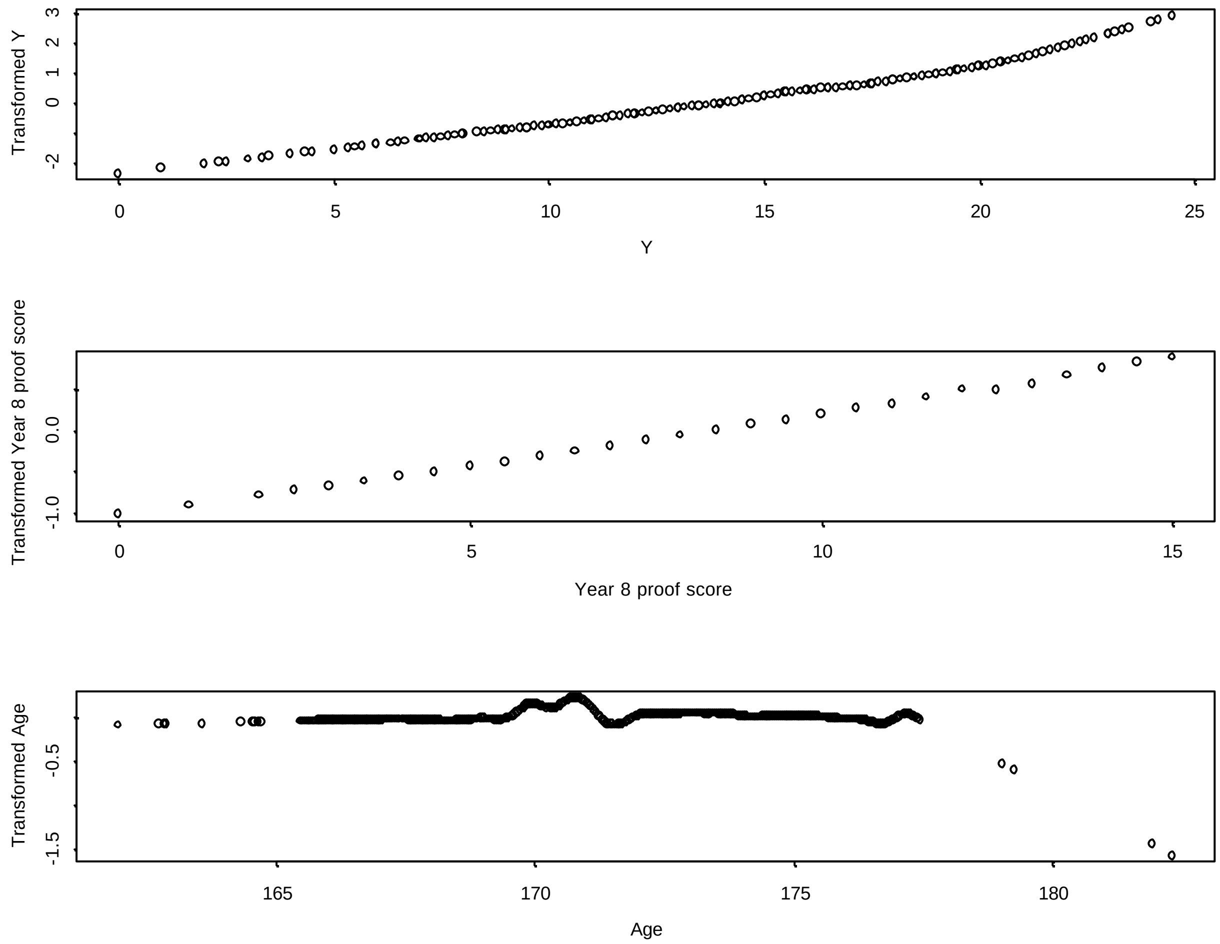
Here we search for efficient transformations using the ACE (Alternating Conditional Expectation) algorithm (Breiman and Friedman 1985; De Veaux 1989), a technique originally developed for finding nonparametric transformations of both the independent and dependent variables in multiple linear regression such that the relationship between them is as linear as possible. The ACE results can be used to indicate if transformations are necessary and, if so, to suggest empirically meaningful parametric or nonparametric transformations for use in logistic regression. However, we do not actually use the non-parametric transformations estimated by ACE in their raw form but find functional approximations to them which we then estimate.

This approach is implemented using the ace and avas functions in the S-PLUS statistical package for multiple linear regression (Venables and Ripley 1994). To apply the ACE algorithm to possibly recode an independent variable, an examination of the scatterplot of the original values against the ACE transformed values may give insight into the nature of an approximately optimal transformation. For example, if the plot is judged linear, a linear model with the original data is appropriate, if not, it suggests an idea of what kind of transformation should be employed. Possibilities include power and log transformations, threshold and piecewise linear functions, categorical (i.e., piecewise constant) transformations, and quadratic or higher order polynomial terms. If the suggested transformations have the form of a threshold effect, where the nature of the relationship between the response variable and covariate changes beyond a particular point threshold value, we may then consider recoding the variable into piecewise linear or sometime categorical variables. It is often possible that there are a number of potential candidates for transformation of a variable. To select the best transformation, we use the *BIC* (Bayesian Information Criterion) because most models considered here are non-nested and this statistic has been demonstrated to be a good index for comparing non-nested models (Raftery 1995). The *BIC* statistic, originally derived from a Bayesian approach to statistical inference, can be expressed as:

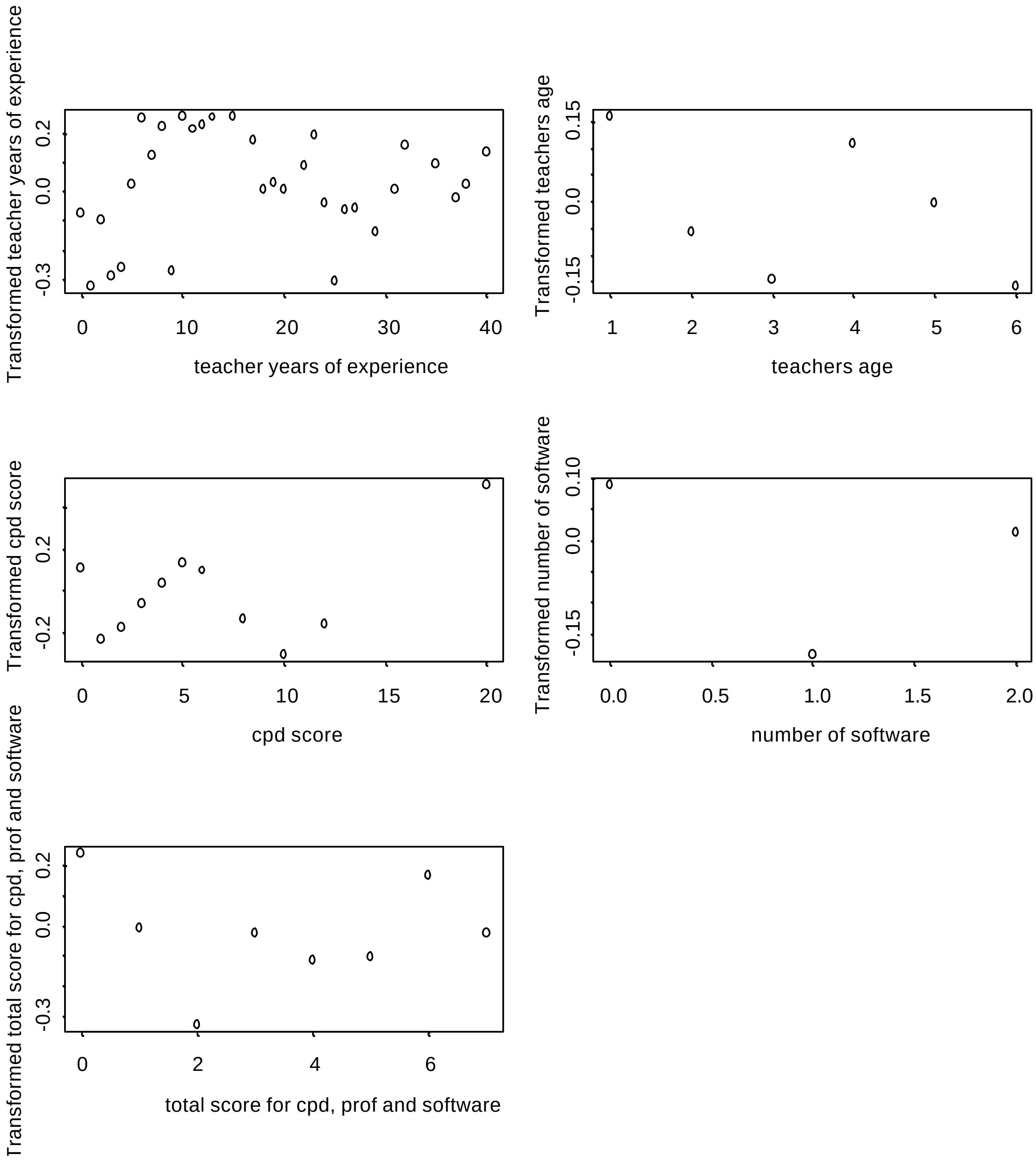
$$BIC = -?2 + p \log N$$

where ?2 is the likelihood ratio test statistic for comparing the null model without covariates with model of interest, p is the number of independent variables in the model of interest and N is the sample size, which in our case is the number of cases in the logistic regression. The smaller *BIC* is, the better the model. However, to test the differences between nested models, we use the conventional likelihood ratio statistic.

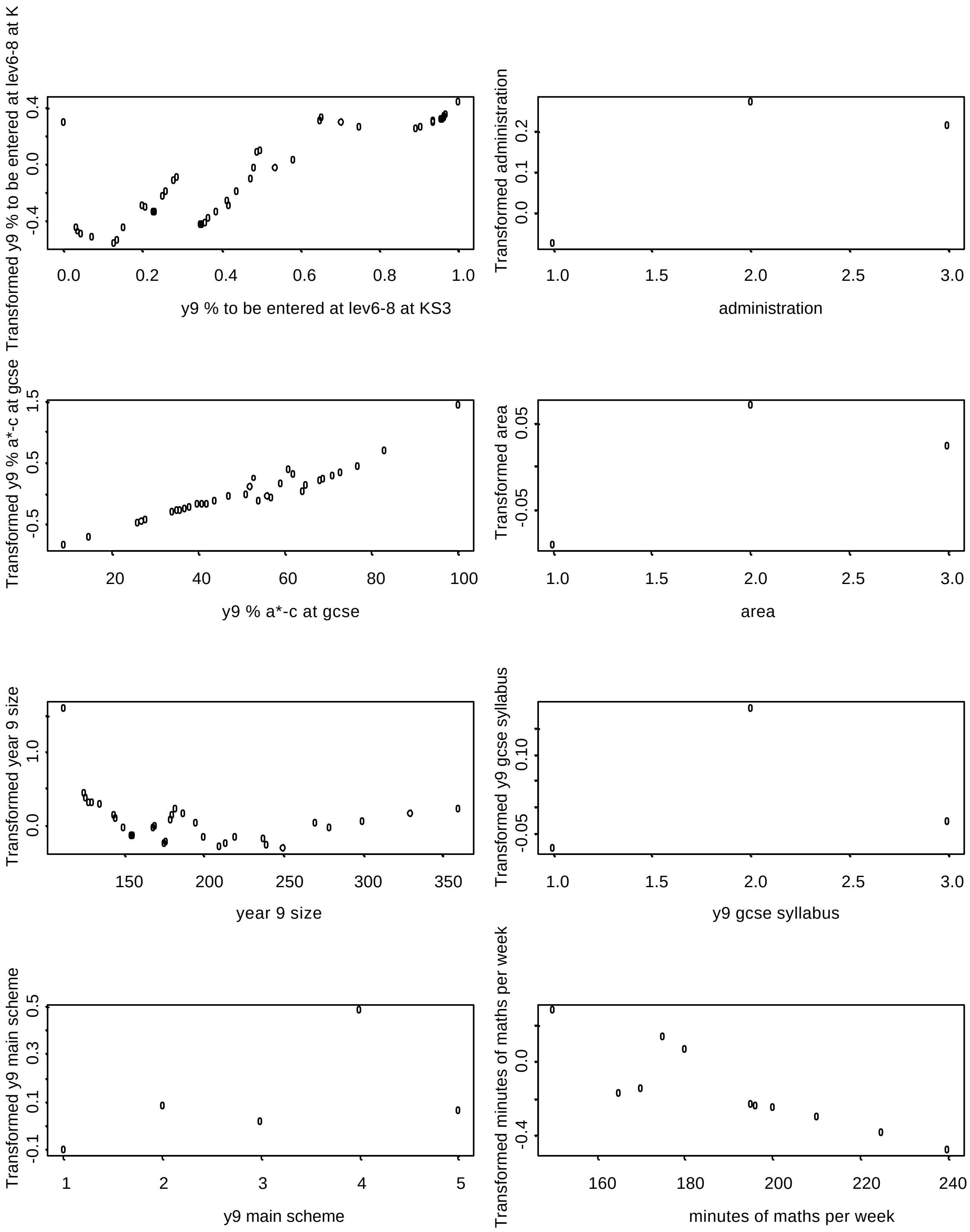
The following ACE plots for various variables are plotted in the following fix pages.



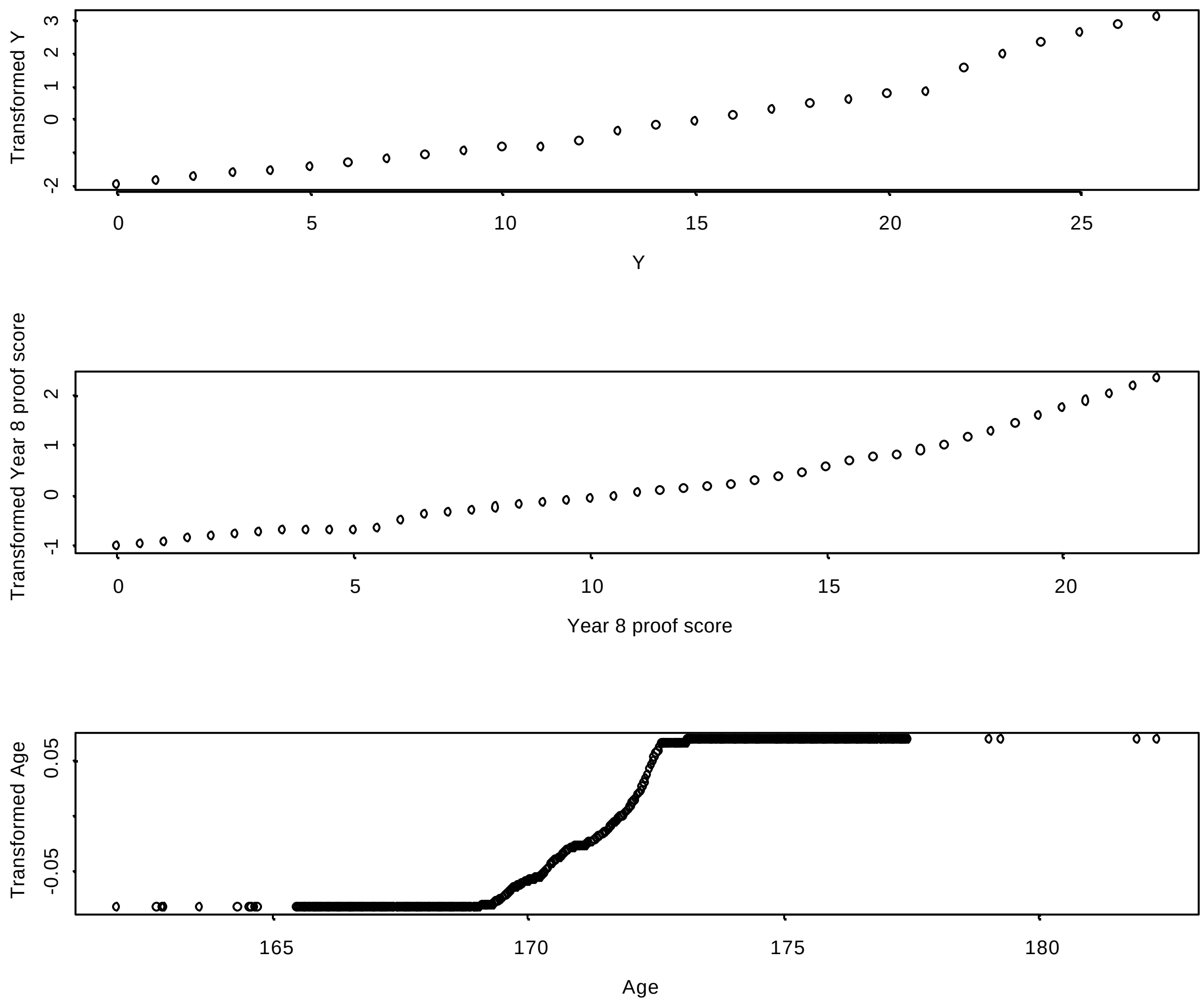
ACE plots for geometry analysis



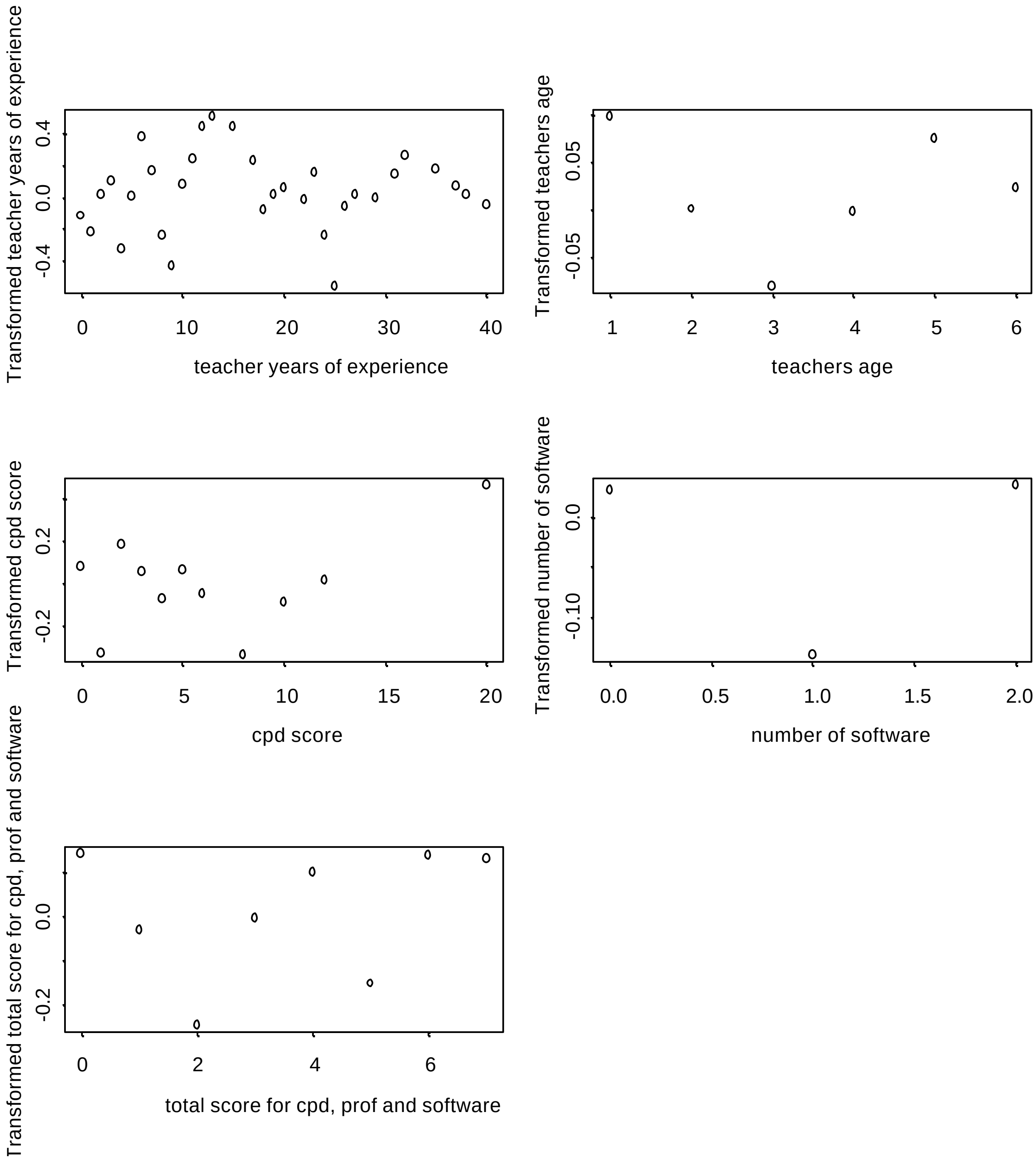
ACE plots for geometry analysis



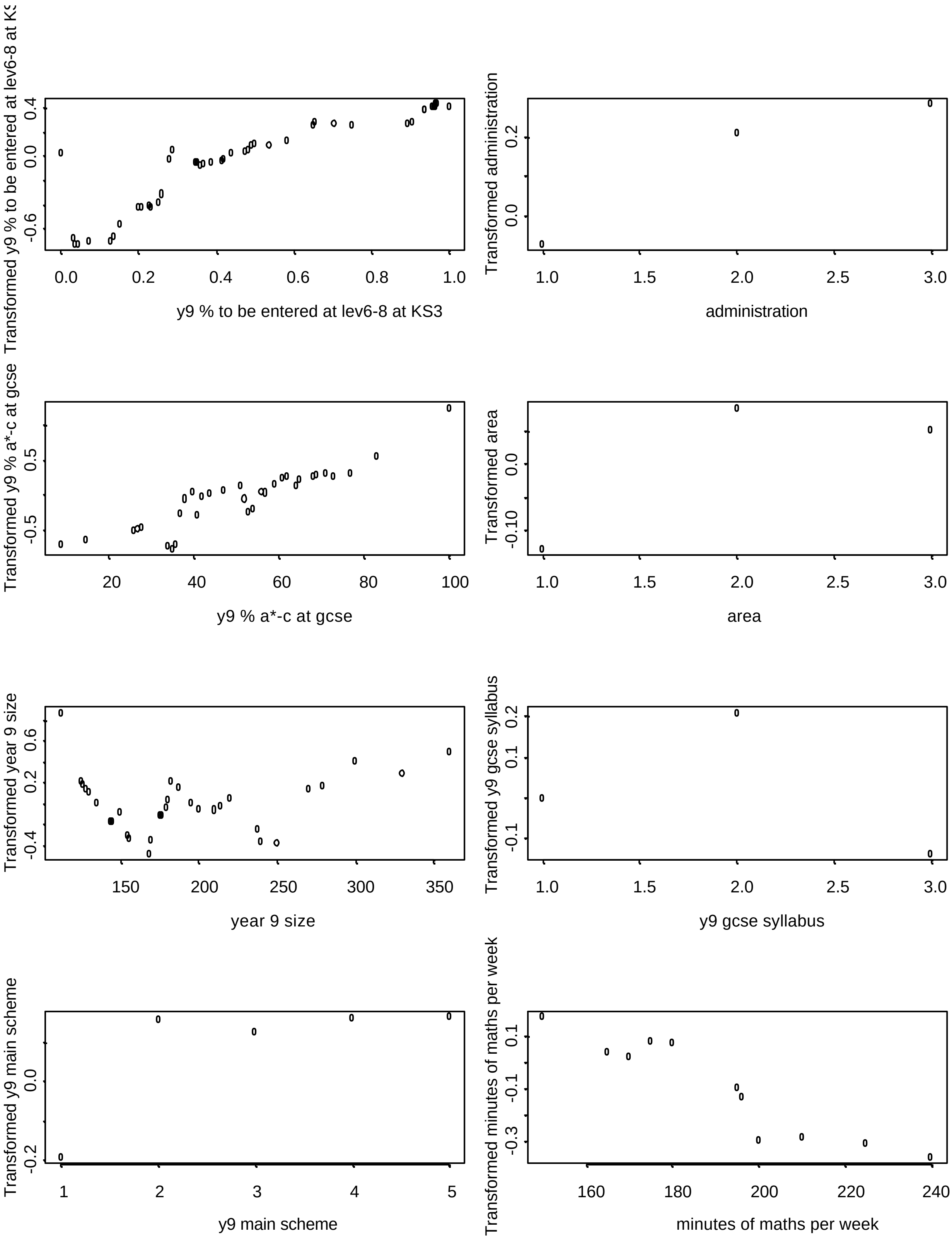
ACE plots for geometry analysis



ACE plots for algebra analysis



ACE plots for algebra analysis



ACE plots for algebra analysis

Based on the ACE plots above and BIC statistic calculation, we decided the following transformation or codings for the variable in the analysis:

Variables	Name	Variable Transformation or Codings
Response Variable		
Year 9 proof score	YR9prf	linear
Predictor Variables		
Year 8 proof score	YR8prf	linear
school-level:		
administration	sadmin	0=county, 1=voluntary (VA/VC) or other
age range 11-16 or 11-18	sage	1=11-16, 0=11-18
% A*-C at GCSE	s%A_C	linear
gender	smix	1=girls only, 0=mixed
area (one of three)	sarea	1=urban or suburban, 0=rural
year-9 size	ssize	linear(school size)
% to be entered at lev6-8 at KS3	spent1	min(Year 9 % enter at KS3, 15%)
	spent2	max(0,min(Year 9 % at KS3-15%,25%-15%))"
	spent3	max(0,Year 9 % enter at KS3-25%)
GCSE syllabus	ssyll	1=OCR,0=Others
maths textbook or scheme in use	stext	1=SMP/Vickers/ST(P)/other,0=Key Math
minutes of maths per week	smtime	linear (minutes)
existence of maths club	smclub	1=Yes,0=No
class-level:		
teacher gender	tsex	1=Female, 0=Male
teacher years of experience	tyearex	tyearex1=min(year,12) tyearex2=max(0,min(year-12,22-12) tyearex3=max(0,year-22)
teacher age	tage	linear(year)
teacher degree	tdegree	1=having a maths degree, 0=otherwise
teacher PGCE or Cert	tpgrt	1=having a PGCE or Cert,0=otherwise
teacher HE	tHE	1=having MSc or PhD degree, 0=otherwise
teacher CPD score	tcpd	linear(score)
teacher membership of a professional assoc.	tmember	1=Yes, 0=No
teacher knowledge/use of software	tsoft	1=Yes,0=No
teacher total score for CPD, PROF and software	ttscore	linear(score)
student-level:		
gender	girl	1=Girl,0=Boy
age in months	age	linear(month)

3.2 Model 0

The fixed-part estimates from this model may be expressed by means of the two equations:

$$\text{predicted geometry score} = 13.49 + 0.2724\,girl,$$

$$\text{predicted algebra score} = 14.30 + 0.410\,girl,$$

thus, the predicted score for the base group (boys) in Geometry is 13.49 (s.e. 0.32) and in Algebra it is 14.3 (s.e. 0.36). The gender coefficients are statistically non-significant, as is shown in the full tabulation below:

parameter	estimate	s. error(u)
alg_cons	14.3	0.3608
alg_girl	0.4103	0.2321
geo_cons	13.49	0.3199
geo_girl	0.2724	0.218

Thus, there is *no* statistically significant gender effect on either outcome score when the model makes no adjustment for baseline score.

The residual variance/covariance matrices for the outcome scores at the three levels, school, class, and student, have the following estimates (covariances have been converted to correlations for convenience):

School level

	Geo	*Alg*
Geo	2.55	
Alg	$r = .85$	4.10

Class level

	Geo	*Alg*
Geo	.2.94	
Alg	$r = .73$	2.72

Student level

	Geo	*Alg*
Geo	20.87	
Alg	$r = .41$	23.67

The full tabulation is:

PARAMETER	ESTIMATE	S. ERROR	CORR.
School			
$\sigma_w^2(\text{geo\_cons})$	2.549	1.154	1
$\sigma_w(\text{geo\_cons, alg\_cons})$	2.752	1.119	0.851
$\sigma_w^2(\text{alg\_cons})$	4.099	1.417	1
Class			
$\sigma_v^2(\text{geo\_cons})$	2.943	0.9846	1
$\sigma_v(\text{geo\_cons, alg\_cons})$	2.072	0.8307	0.733
$\sigma_v^2(\text{alg\_cons})$	2.717	0.9853	1
Student			
$\sigma_u^2(\text{geo\_cons})$	20.87	0.6835	1
$\sigma_u(\text{geo\_cons, alg\_cons})$	9.146	0.5569	0.412
$\sigma_u^2(\text{alg\_cons})$	23.67	0.7757	1

Two important observations can be found from the above results. First, there is statistically significant, though small, residual variation at class level, and high residual correlation ($r \approx .85$ and 0.73) at school and class level between the two outcomes. Thus, in this very simple model, schools that perform above the average in algebra are predicted to do so in geometry also, and vice versa.

Second, class effects on the two subjects, within schools, are predicted to be similar. A student, however, who performs above the expectation for her class in algebra has only a slight tendency to perform above expectation in geometry also. In percentages, the residual variances at school, class, and student levels, are in the ratio 2.55:2.94:20.87 for Geometry and 4.10:2.72:23.67 for Algebra.

3.3 Model 1

We have included the YR8prf in this model as a predictor. Once YR8prf score is included, no statistically significant variation remains at class level within school. Thus, we have two levels of variation, students at level 2 within schools at level 3, with level 1 used as before to distinguish between the responses in Algebra and Geometry.

In this model, the fixed-part predictors are gender and YR8prf score. The fixed part of the model for geometry scores is estimated to be:

$$\text{predicted geometry score} = 9.14 + 0.52YR8prf + 0.39girl,$$

a linear relationship with the baseline score, with an additional predicted benefit of 0.39 raw-score point for the girls over the boys.

The corresponding model for algebra scores is

$$\text{predicted algebra score} = 8.886 + 0.53YR8prf + .67girl,$$

also a linear relationship with the YR8prf score with, this time, an additional predicted benefit of 0.67 raw-score point for the girls over the boys.

Please note that the effect of gender on the proof score is not significant in either of the two equations.

The full tabulation of the fixed part, showing the standard errors, is:

parameter	estimate	s. error
alg_cons	8.886	0.3449
alg_girl	0.6662	0.3626
alg_YR8prf	0.5261	0.02496
geo_cons	9.144	0.3776
geo_girl	0.3897	0.2661
geo_YR8prf	0.5196	0.0306

The relationship described by each equation is illustrated in the first of the two graphs on the following pages. The second graph shows the relationship of the outcome scores with the raw baseline score.

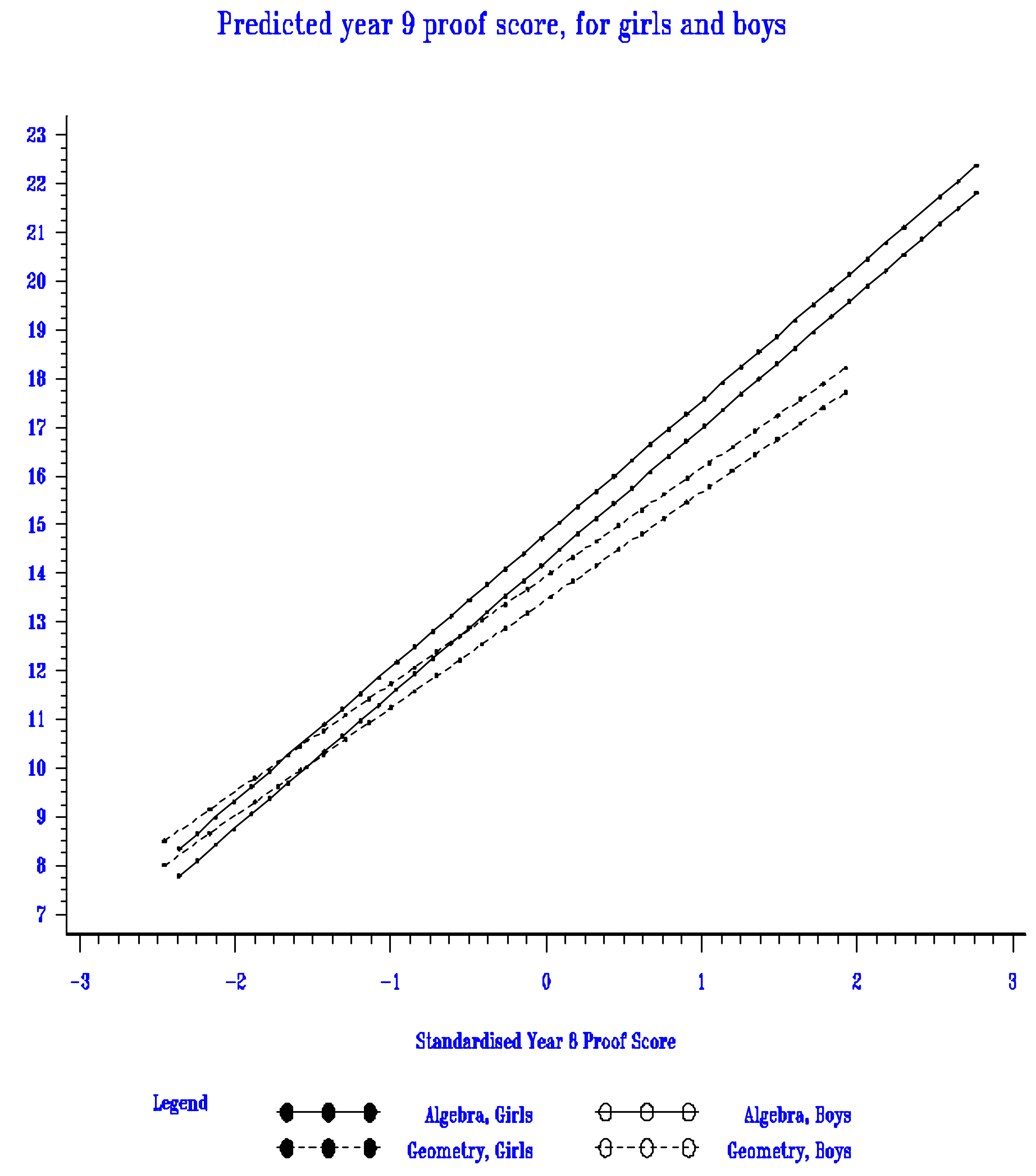
The random part of Model 1 can be expressed as follows:

School level (variances on the diagonal; correlations elsewhere)

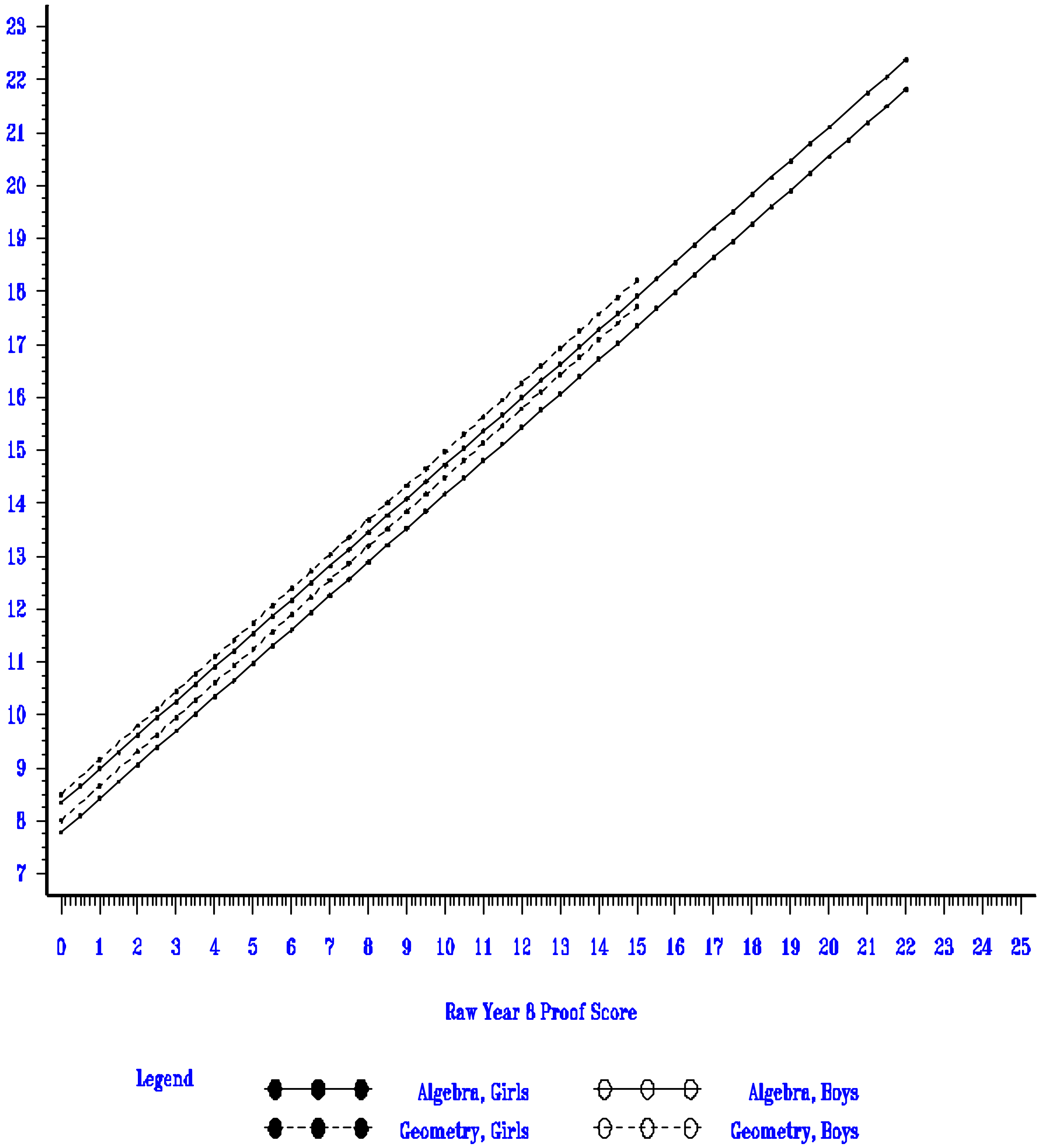
		Girls		*Boys*	
		Geo	*Alg*	*Geo*	*Alg*
Girls	*Geo*	2.137			
	Alg	$r = .41$	3.191		
Boys	*Geo*	$r = .70$	$r = .41$	3.014	
	Alg			$r = .53$	1.687

Student level (variances on the diagonal; correlations elsewhere)

		Girls		*Boys*	
		Geo	*Alg*	*Geo*	*Alg*
Girls	*Geo*	18.54			
	Alg	$r = .28$	19.66		
Boys	*Geo*			18.87	
	Alg				21.00


Predicted year 9 proof score, for girls and boys
23
22
21
20
19
18
17
16
15
14
13
12
11
10
9
8
7
-3
-2
-1
0
1
2
3
Standardised Year 8 Proof Score
Legend
Algebra, Girls
Geometry, Girls
Algebra, Boys
Geometry, Boys

Predicted year 9 proof score, for girls and boys



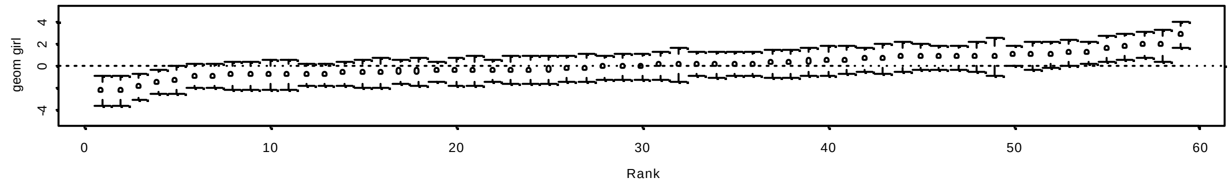
The full tabulation is:

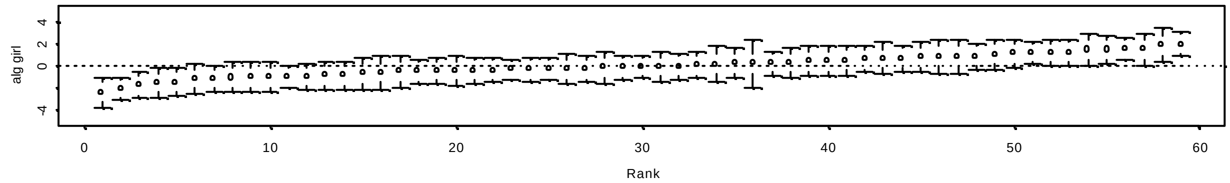
PARAMETER	ESTIMATE	S. ERROR	CORR.
School			
$\sigma_v^2(\text{geo\_girl})$	2.137	0.5892	1
$\sigma_v(\text{alg\_girl, geo\_girl})$	1.076	0.4725	0.41
$\sigma_v^2(\text{alg\_girl})$	3.191	0.8283	1
$\sigma_v(\text{geo\_boy, geo\_girl})$	1.781	0.4931	0.702
$\sigma_v^2(\text{geo\_boy})$	3.014	0.769	1
$\sigma_v(\text{alg\_boy, geo\_boy})$	1.186	0.4356	0.526
$\sigma_v^2(\text{alg\_boy})$	1.687	0.5649	1
Student			
$\sigma_u^2(\text{geo\_girl})$	18.54	0.859	1
$\sigma_u(\text{alg\_girl, geo\_girl})$	5.283	0.6499	0.277
$\sigma_u^2(\text{alg\_girl})$	19.66	0.9127	1
$\sigma_u^2(\text{geo\_boy})$	18.87	0.8825	1
$\sigma_u^2(\text{alg\_boy})$	21.00	0.9813	1

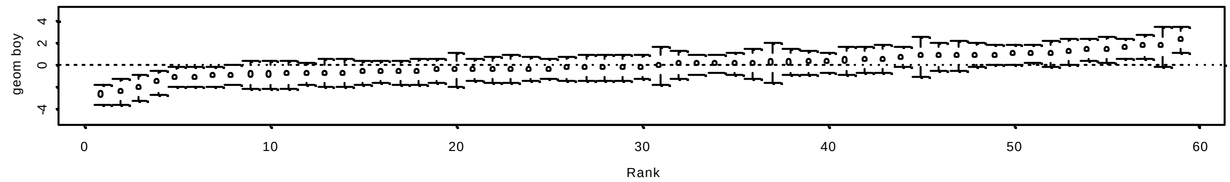
The high value of correlation between girls' and boys' scores (within subject) at school level in geometry means that, for example, a school that has a high residual for boys is very likely to have a high residual also for girls in the same subject.

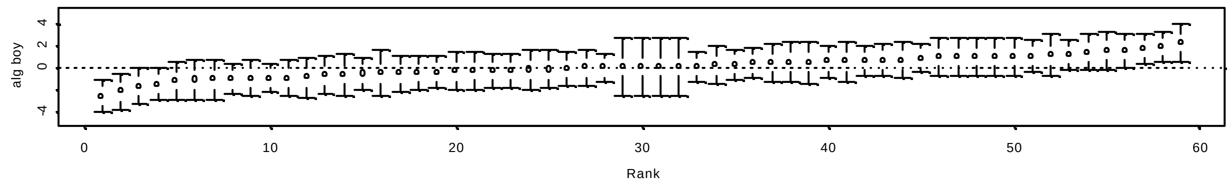
It appears from the tables that schools are slightly less variable in their boys' residual performance at algebra (compared to predictions) than in their girls' performance: but this difference does not reach statistical significance. Elsewhere, the variances of boys and girls within subject are similar. A correlation between boys and girls is not meaningful at student level (i.e. within a student), and correlations within students between their residual performance in algebra and in geometry (compared to predictions) are weak. A student who performs better than expected for their school in, say, algebra (after adjustment for their gender and baseline score and the school's residual) is not especially likely to perform better than expected for their school in geometry also.

The following chart plots school-level residuals against their ranks, with error bars corresponding to 1.96 SD. Thus, an error bar wholly above the dotted line corresponds to a school that is performing above the mean predicted by the model, with 95% confidence.









There are 59 schools in the sample, and all have girl students. Four of the schools are girls-only, hence only 55 schools feature in the chart for boys' residuals.

Tables of the school residual ranks now follow (A high-numbered rank indicates good performance.)

School Residual Ranks from Model 1

School	geo_girl	alg_girl	alg_boy	geo_boy
1	55	56	.	.
2	37	43	44	30
3	31	21	20	26
4	12	28	38	21
5	33	36	.	.
6	18	20	11	8
7	36	44	47	36
8	27	38	43	17
9	56	4	9	54
10	21	40	30	27
11	28	3	2	23
12	19	23	33	19
13	39	49	12	32
14	9	2	4	12
15	24	8	7	31
16	15	9	6	14
17	52	53	51	50
18	16	17	23	18
20	51	26	54	51
21	44	33	32	46
22	25	14	18	28
23	4	19	28	4
24	30	54	46	33
25	46	27	37	45
26	38	48	36	38
27	58	58	41	52
28	54	52	45	47
29	1	42	48	3
30	59	59	.	.
31	47	50	.	.
32	8	13	29	7
33	48	32	22	39
34	50	24	14	48
35	29	45	24	5
36	5	1	5	22
37	26	10	17	35
39	32	34	21	16
40	17	7	10	11
41	34	12	3	25
42	13	6	8	15
43	45	55	40	42
44	22	18	13	9
45	53	31	25	49
46	11	15	16	10
48	57	51	55	55
50	2	46	35	2
51	42	41	26	37
52	6	30	15	6
53	23	35	39	20
54	3	5	1	1
55	20	16	49	40
56	43	47	34	41
57	35	22	27	34
58	7	11	19	13
59	14	25	52	43
60	49	37	31	44
61	10	39	42	24
62	41	29	53	53
64	40	57	50	29

Schools ranked according to their residuals for girls geometry (Model 1)

School	geo_girl	alg_girl	alg_boy	geo_boy
30	59	59	.	.
27	58	58	41	52
48	57	51	55	55
9	56	4	9	54
1	55	56	.	.
28	54	52	45	47
45	53	31	25	49
17	52	53	51	50
20	51	26	54	51
34	50	24	14	48
60	49	37	31	44
33	48	32	22	39
31	47	50	.	.
25	46	27	37	45
43	45	55	40	42
21	44	33	32	46
56	43	47	34	41
51	42	41	26	37
62	41	29	53	53
64	40	57	50	29
13	39	49	12	32
26	38	48	36	38
2	37	43	44	30
7	36	44	47	36
57	35	22	27	34
41	34	12	3	25
5	33	36	.	.
39	32	34	21	16
3	31	21	20	26
24	30	54	46	33
35	29	45	24	5
11	28	3	2	23
8	27	38	43	17
37	26	10	17	35
22	25	14	18	28
15	24	8	7	31
53	23	35	39	20
44	22	18	13	9
10	21	40	30	27
55	20	16	49	40
12	19	23	33	19
6	18	20	11	8
40	17	7	10	11
18	16	17	23	18
16	15	9	6	14
59	14	25	52	43
42	13	6	8	15
4	12	28	38	21
46	11	15	16	10
61	10	39	42	24
14	9	2	4	12
32	8	13	29	7
58	7	11	19	13
52	6	30	15	6
36	5	1	5	22
23	4	19	28	4
54	3	5	1	1
50	2	46	35	2
29	1	42	48	3

Schools ranked according to their residuals for girls algebra (Model 1)

School	geo_girl	alg_girl	alg_boy	geo_boy
30	59	59	.	.
27	58	58	41	52
64	40	57	50	29
1	55	56	.	.
43	45	55	40	42
24	30	54	46	33
17	52	53	51	50
28	54	52	45	47
48	57	51	55	55
31	47	50	.	.
13	39	49	12	32
26	38	48	36	38
56	43	47	34	41
50	2	46	35	2
35	29	45	24	5
7	36	44	47	36
2	37	43	44	30
29	1	42	48	3
51	42	41	26	37
10	21	40	30	27
61	10	39	42	24
8	27	38	43	17
60	49	37	31	44
5	33	36	.	.
53	23	35	39	20
39	32	34	21	16
21	44	33	32	46
33	48	32	22	39
45	53	31	25	49
52	6	30	15	6
62	41	29	53	53
4	12	28	38	21
25	46	27	37	45
20	51	26	54	51
59	14	25	52	43
34	50	24	14	48
12	19	23	33	19
57	35	22	27	34
3	31	21	20	26
6	18	20	11	8
23	4	19	28	4
44	22	18	13	9
18	16	17	23	18
55	20	16	49	40
46	11	15	16	10
22	25	14	18	28
32	8	13	29	7
41	34	12	3	25
58	7	11	19	13
37	26	10	17	35
16	15	9	6	14
15	24	8	7	31
40	17	7	10	11
42	13	6	8	15
54	3	5	1	1
9	56	4	9	54
11	28	3	2	23
14	9	2	4	12
36	5	1	5	22

School Outliers according to geometry

NAME	COL1	COL2	COL3	COL4	COL5	COL6	COL7	COL8
Above the mean for girls geometry	30	27	48	9	1	45	.	.
Above the mean for boys geometry	48	9	62	27	20	45	21	.
Below the mean for girls geometry	58	36	23	54	50	29	.	.
Below the mean for boys geometry	44	6	52	35	23	29	50	54

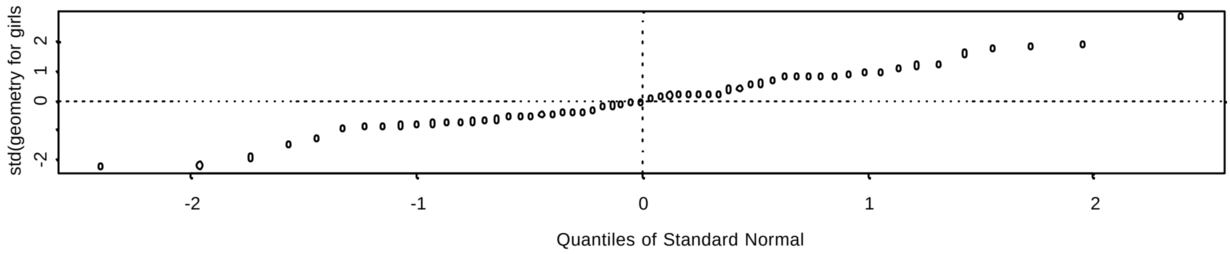
School Outliers according to algebra

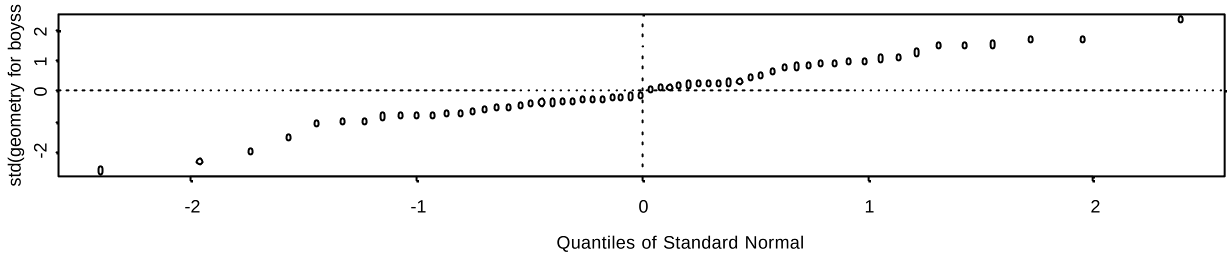
NAME	COL1	COL2	COL3	COL4	COL5	COL6	COL7	COL8	COL9	COL10
Above the mean for girls algebra	30	27	64	1	43	24	17	28	48	31
Above the mean for boys algebra	48	20	62	59	.	.	.	.	.	.
Below the mean for girls algebra	42	54	9	11	14	36	.	.	.	.
Below the mean for boys algebra	14	11	54	.	.	.	.	.	.	.

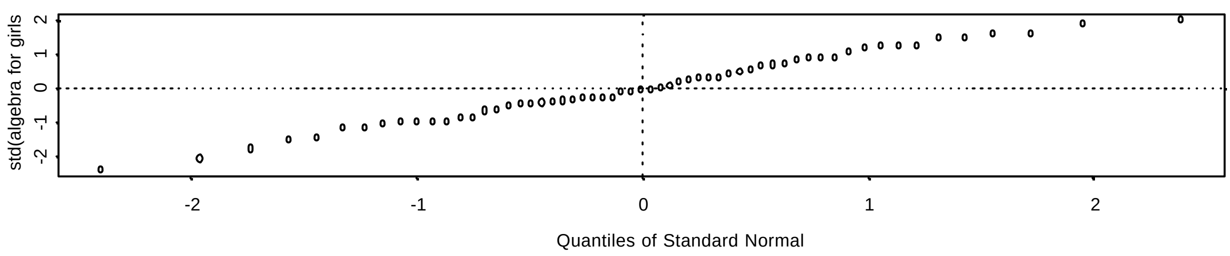
School Outliers according to difference between geometry and algebra

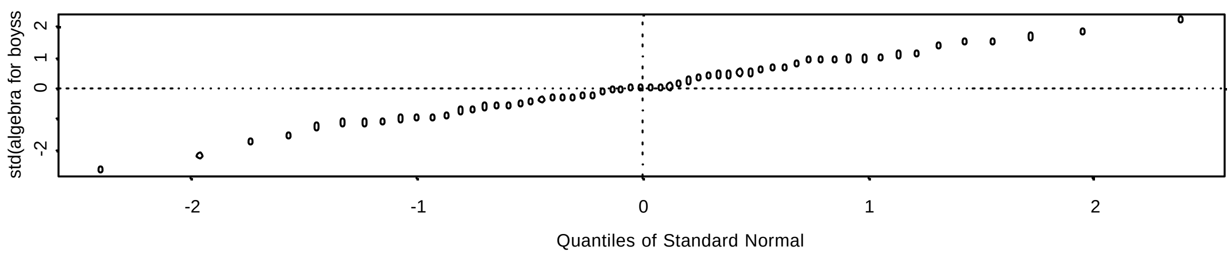
NAME	COL1	COL2	COL3	COL4	COL5
Large differences between rankings on girls geometry and algebra	9	11	14	29	50
Large differences between rankings on boys geometry and algebra	9	34	29	23	50

We also plotted standardised diagnostic school-level residuals against their normal scores (see below). These are not ideal, and reflect problems in the scoring of the outcome.


std(geometry for girls
Quantiles of Standard Normal


std(geometry for boyss
Quantiles of Standard Normal


std(algebra for girls
Quantiles of Standard Normal


std(algebra for boyss
Quantiles of Standard Normal

3.4 Model 2

Model 2 is the most parsimonious model selected by forward and backward procedure.

The fixed part of Model 2 is:

$$\text{predicted geometry score} = 6.57 + 0.46 YR8prf + 0.32 girl + 0.06 \times s\%A\_C,$$
$$\text{predicted algebra score} = 8.56 + 0.50 YR8prf + 0.71 girl + 1.13 tHE + 1.22 \times stext,$$

The fixed-part parameters are tabulated, with their standard errors, on the next page.

parameter	estimate	s. error(u)
alg_cons	8.564	0.3949
alg_girl	0.7057	0.3567
alg-YR8prf	0.4797	0.02466
alg_tHE	1.131	0.4736
alg_stext	1.22	0.3584
geo_cons	6.568	0.6457
geo_girl	0.317	0.2409
geo_YR8prf	0.4617	0.03035
geo_s%A_C	0.05988	0.01119

One variable s%A_C (the school % GCSE pass rate at A*-C), a historical measure which might be regarded as a compositional effect on the sample students' performance in proof. The other effects in this model, including school-level residuals, are conditional on this effect. Thus, if this model is used to compare schools, the schools are being compared for the additional effects they are having on students' performance in proof, over and above what may underlie historical performance at GCSE.

The results show that s%A_C is statistically significantly associated with the geometry score: 10% increase in this variable will result in .6 proof score.

Two additional variables are found to be significant predictors of proof algebra score. A teacher with a MSc or PhD degree education will on average increase the proof score by 1.131 and students using of textbooks other than "Key Math" will have 1.22 higher proof score than those using other textbooks.

Please note that effect of gender is significant only for algebra but not for geometry.

Turning to the random part of Model 2, we find that including the additional predictor in the fixed part reduces the school-level variation. Schools' residual performance in algebra is significantly more variable for girls than for boys. There is little change from Model 1 in the random part at student level.

School level (variances on the diagonal; correlations elsewhere)

		Girls		*Boys*	
		Geo	*Alg*	*Geo*	*Alg*
Girls	*Geo*	1.046			
	Alg	$r = .833$	2.73		
Boys	*Geo*			2.538	
	Alg				1.871

Student level (variances on the diagonal; correlations elsewhere)

		Girls		*Boys*	
		Geo	*Alg*	*Geo*	*Alg*
Girls	*Geo*	18.64			
	Alg	$r = .29$	19.58		
Boys	*Geo*			18.88	
	Alg			0.29	21.00

The full tabulation of the random part of Model 2 is:

PARAMETER	ESTIMATE	S. ERROR	CORR.
School			
$\sigma_v^2(\text{geo\_girl})$	1.046	0.398	1
$\sigma_v^2(\text{alg\_girl})$	2.73	0.7277	1
$\sigma_v(\text{geo\_boy, geo\_girl})$	1.357	0.4188	0.833
$s_v(\text{geo\_boy})$	2.538	0.6997	1
$\sigma_v^2(\text{alg\_boy})$	1.871	0.5969	1
Student			
$\sigma_u^2(\text{geo\_girl})$	18.64	0.8621	1
$\sigma_u(\text{alg\_girl, geo\_girl})$	5.567	0.6484	0.291
$\sigma_u^2(\text{alg\_girl})$	19.58	0.908	1
$\sigma_u^2(\text{geo\_boy})$	18.88	0.8827	1
$\sigma_u(\text{alg\_boy, geo\_boy})$	5.847	0.6833	0.294
$\sigma_u^2(\text{alg\_boy})$	21.00	0.9813	1

School Residual Ranks from Model 1

School	geo_girl	alg_girl	alg_boy	geo_boy
1	42	52	.	.
2	38	30	30	31
3	14	13	19	9
4	16	29	31	22
5	44	42	.	.
6	6	23	16	6
7	22	41	43	20
8	24	25	35	17
9	59	1	4	55
10	7	31	14	16
11	51	4	3	40
12	27	32	46	25
13	20	48	5	29
14	26	3	10	26
15	41	21	7	41
16	28	17	8	34
17	50	58	52	47
18	10	19	20	10
20	52	16	51	43
21	40	27	25	38
22	35	9	13	36
23	3	12	32	3
24	18	54	40	28
25	37	34	47	33
26	12	45	29	13
27	55	57	33	52
28	47	47	39	42
29	4	50	55	4
30	57	51	.	.
31	36	59	.	.
32	25	14	44	14
33	32	36	34	24
34	53	22	6	51
35	31	40	24	12
36	43	2	2	48
37	46	15	21	46
39	34	35	36	23
40	39	8	18	32
41	13	11	12	8
42	21	7	9	27
43	29	55	28	37
44	11	26	23	7
45	58	38	38	50
46	17	10	17	18
48	56	46	53	53
50	2	56	49	2
51	49	37	22	44
52	9	28	11	11
53	5	43	48	5
54	1	5	1	1
55	19	20	42	30
56	33	44	27	35
57	45	18	37	39
58	15	6	15	21
59	30	24	41	49
60	48	33	26	45
61	8	49	45	15
62	54	39	54	54
64	23	53	50	19

Schools ranked according to their residuals for girls geometry (Model 1)

School	geo_girl	alg_girl	alg_boy	geo_boy
9	59	1	4	55
45	58	38	38	50
30	57	51	.	.
48	56	46	53	53
27	55	57	33	52
62	54	39	54	54
34	53	22	6	51
20	52	16	51	43
11	51	4	3	40
17	50	58	52	47
51	49	37	22	44
60	48	33	26	45
28	47	47	39	42
37	46	15	21	46
57	45	18	37	39
5	44	42	.	.
36	43	2	2	48
1	42	52	.	.
15	41	21	7	41
21	40	27	25	38
40	39	8	18	32
2	38	30	30	31
25	37	34	47	33
31	36	59	.	.
22	35	9	13	36
39	34	35	36	23
56	33	44	27	35
33	32	36	34	24
35	31	40	24	12
59	30	24	41	49
43	29	55	28	37
16	28	17	8	34
12	27	32	46	25
14	26	3	10	26
32	25	14	44	14
8	24	25	35	17
64	23	53	50	19
7	22	41	43	20
42	21	7	9	27
13	20	48	5	29
55	19	20	42	30
24	18	54	40	28
46	17	10	17	18
4	16	29	31	22
58	15	6	15	21
3	14	13	19	9
41	13	11	12	8
26	12	45	29	13
44	11	26	23	7
18	10	19	20	10
52	9	28	11	11
61	8	49	45	15
10	7	31	14	16
6	6	23	16	6
53	5	43	48	5
29	4	50	55	4
23	3	12	32	3
50	2	56	49	2
54	1	5	1	1

Schools ranked according to their residuals for girls algebra (Model 1)

School	geo_girl	alg_girl	alg_boy	geo_boy
31	36	59	.	.
17	50	58	52	47
27	55	57	33	52
50	2	56	49	2
43	29	55	28	37
24	18	54	40	28
64	23	53	50	19
1	42	52	.	.
30	57	51	.	.
29	4	50	55	4
61	8	49	45	15
13	20	48	5	29
28	47	47	39	42
48	56	46	53	53
26	12	45	29	13
56	33	44	27	35
53	5	43	48	5
5	44	42	.	.
7	22	41	43	20
35	31	40	24	12
62	54	39	54	54
45	58	38	38	50
51	49	37	22	44
33	32	36	34	24
39	34	35	36	23
25	37	34	47	33
60	48	33	26	45
12	27	32	46	25
10	7	31	14	16
2	38	30	30	31
4	16	29	31	22
52	9	28	11	11
21	40	27	25	38
44	11	26	23	7
8	24	25	35	17
59	30	24	41	49
6	6	23	16	6
34	53	22	6	51
15	41	21	7	41
55	19	20	42	30
18	10	19	20	10
57	45	18	37	39
16	28	17	8	34
20	52	16	51	43
37	46	15	21	46
32	25	14	44	14
3	14	13	19	9
23	3	12	32	3
41	13	11	12	8
46	17	10	17	18
22	35	9	13	36
40	39	8	18	32
42	21	7	9	27
58	15	6	15	21
54	1	5	1	1
11	51	4	3	40
14	26	3	10	26
36	43	2	2	48
9	59	1	4	55

School Outliers according to geometry

NAME	COL1	COL2	COL3	COL4	COL5	COL6
Above the mean for girls geometry	9	45	.	.	.	.
Above the mean for boys geometry	9	62	48	34	45	59
Below the mean for girls geometry	23	50	54	.	.	.
Below the mean for boys geometry	44	6	29	23	50	54

School Outliers according to algebra

NAME	COL1	COL2	COL3	COL4	COL5	COL6
Above the mean for girls algebra	31	17	50	24	1	.
Above the mean for boys algebra	29	62	.	.	.	.
Below the mean for girls algebra	58	54	11	14	36	9
Below the mean for boys algebra	54	.	.	.	.	.

School Outliers according to difference between geometry and algebra

NAME	COL1	COL2	COL3	COL4	COL5
Large differences between rankings on girls geometry and algebra	9	11	36	14	20
Large differences between rankings on boys geometry and algebra	36	34	50	53	29

NAME	COL6	COL7	COL8	COL9
Large differences between rankings on girls geometry and algebra	31	24	29	50
Large differences between rankings on boys geometry and algebra	9	23	11	.

Largest changes in rankings from model 1 to model 2:

School ID	Change in geo_girl	School ID	Change in alg_girl	School ID	Change in geo_boy	School ID	Change in alg_boy
26	-26	8	-13	26	-25	10	-16
41	-21	2	-13	3	-17	2	-14
13	-19	20	-10	41	-17	43	-12
40	22	50	10	16	20	50	14
11	23	62	10	40	21	39	15
36	38	15	13	36	26	32	15

The next two models are included to illustrate what happens when the random effect at school level is removed from gender and attached instead to the intercept term, which is common to both boys and girls. This is equivalent to pooling schools' performances for their boys and their girls, in other words assuming their 'effects' are the same for either gender. The fixed part changes very little, but a slightly different ranking of schools arises.

3.5 Model 3

Model 3 (as Model 1, but with no random effect of gender at school level)

The model for the fixed part is:

$$\text{predicted geometry score} = 9.165 + 0.47 YR8prf + 0.31 girl \text{ ,}$$
$$\text{predicted algebra score} = 9.69 + 0.46 YR8prf + .45 girl \text{,}$$

```
parameter            estimate     s. error(u)
alg_cons                9.694       0.3741
alg_girl               0.4491       0.2105
alg_YR8prf             0.4623      0.02472
geo_cons                9.653       0.3647
geo_girl               0.3077       0.2034
geo_YR8prf             0.4673      0.03014
```

The estimated residual variance/correlation matrix at school level is

	Geo	*Alg*
Geo	2.668	
Alg	$r = .509$	3.05

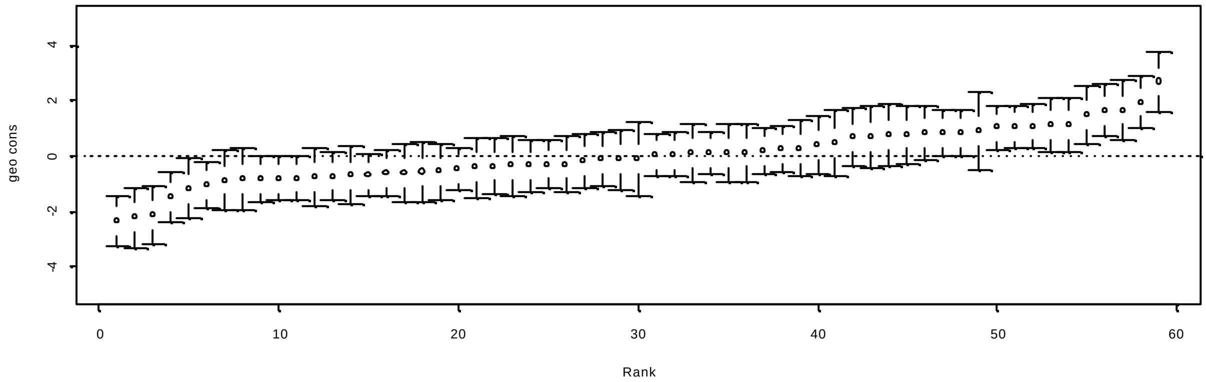
,

Student level (variances on the diagonal; correlations elsewhere)

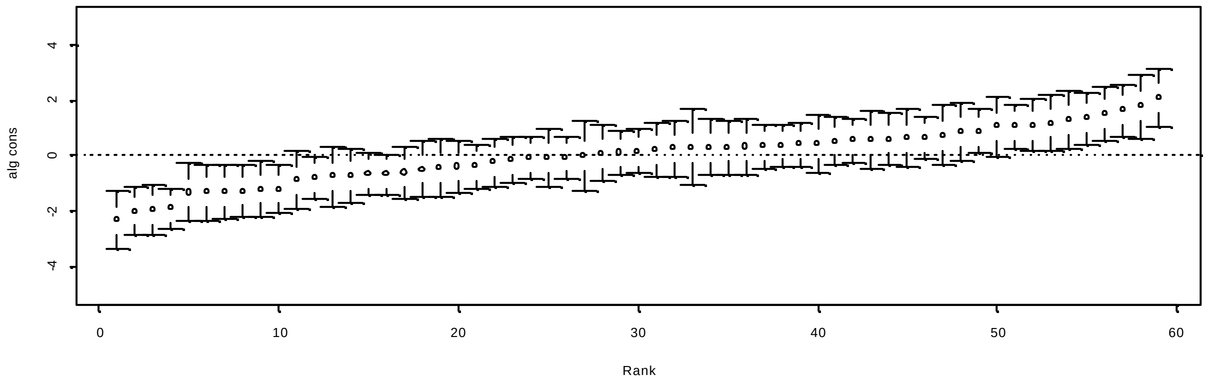
		Girls		*Boys*	
		Geo	*Alg*	*Geo*	*Alg*
Girls	*Geo*	18.71			
	Alg	$r = .29$	19.60		
Boys	*Geo*			19.17	
	Alg			.29	20.97

which compares in an obvious way with that for Model 1. The residual variance/correlation matrix at student level is almost unchanged from Model 1. The full tabulation of the random part follows:

PARAMETER	ESTIMATE	S. ERROR	CORR.
School			
$\sigma_v^2(\text{geo\_cons})$	2.668	0.6086	1
$\sigma_v(\text{geo\_cons, alg\_cons})$	1.452	0.5052	0.509
$\sigma_v^2(\text{alg\_cons})$	3.050	0.6875	1
Student			
$\sigma_u^2(\text{geo\_girl})$	18.71	0.8596	1
$\sigma_u(\text{alg\_girl, geo\_girl})$	5.551	0.6478	0.290
$\sigma_u^2(\text{alg\_girl})$	19.60	0.9009	1
$\sigma_u^2(\text{geo\_boy})$	19.17	0.8886	1
$\sigma_u(\text{alg\_boy, geo\_boy})$	5.791	0.684	0.289
$\sigma_u^2(\text{alg\_boy})$	20.97	0.9714	1



geo cons
4
2
0
-2
-4
0
10
20
30
40
50
60
Rank



alg cons
4
2
0
-2
-4
0
10
20
30
40
50
60
Rank

School Residual Ranks from Model 3

School	Geometry	Algebra
1	55	56
2	34	46
3	28	20
4	19	31
5	36	34
6	11	15
7	38	49
8	25	41
9	56	8
10	22	35
11	24	3
12	20	24
13	35	28
14	10	4
15	21	5
16	12	6
17	54	55
18	17	19
20	52	39
21	47	30
22	26	14
23	4	22
24	32	51
25	48	29
26	39	44
27	57	54
28	53	52
29	3	48
30	59	59
31	45	53
32	7	18
33	42	25
34	46	17
35	16	37
36	5	1
37	29	11
39	30	27
40	14	9
41	27	7
42	13	10
43	44	50
44	15	16
45	51	26
46	8	13
48	58	57
50	2	45
51	40	32
52	6	21
53	23	40
54	1	2
55	33	36
56	43	47
57	37	23
58	9	12
59	31	38
60	49	33
61	18	43
62	50	42
64	41	58

Schools ranked according to their residuals for geometry (Model 3)

School	Geometry	Algebra
30	59	59
48	58	57
27	57	54
9	56	8
1	55	56
17	54	55
28	53	52
20	52	39
45	51	26
62	50	42
60	49	33
25	48	29
21	47	30
34	46	17
31	45	53
43	44	50
56	43	47
33	42	25
64	41	58
51	40	32
26	39	44
7	38	49
57	37	23
5	36	34
13	35	28
2	34	46
55	33	36
24	32	51
59	31	38
39	30	27
37	29	11
3	28	20
41	27	7
22	26	14
8	25	41
11	24	3
53	23	40
10	22	35
15	21	5
12	20	24
4	19	31
61	18	43
18	17	19
35	16	37
44	15	16
40	14	9
42	13	10
16	12	6
6	11	15
14	10	4
58	9	12
46	8	13
32	7	18
52	6	21
36	5	1
23	4	22
29	3	48
50	2	45
54	1	2

Schools ranked according to their residuals for algebra (Model 3)

School	Geometry	Algebra
30	59	59
64	41	58
48	58	57
1	55	56
17	54	55
27	57	54
31	45	53
28	53	52
24	32	51
43	44	50
7	38	49
29	3	48
56	43	47
2	34	46
50	2	45
26	39	44
61	18	43
62	50	42
8	25	41
53	23	40
20	52	39
59	31	38
35	16	37
55	33	36
10	22	35
5	36	34
60	49	33
51	40	32
4	19	31
21	47	30
25	48	29
13	35	28
39	30	27
45	51	26
33	42	25
12	20	24
57	37	23
23	4	22
52	6	21
3	28	20
18	17	19
32	7	18
34	46	17
44	15	16
6	11	15
22	26	14
46	8	13
58	9	12
37	29	11
42	13	10
40	14	9
9	56	8
41	27	7
16	12	6
15	21	5
14	10	4
11	24	3
54	1	2
36	5	1

School Outliers according to geometry

NAME	COL1	COL2	COL3	COL4	COL5	COL6	COL7	COL8	COL9	COL10	COL11
Above the mean for geometry	30	48	27	9	1	17	28	20	45	62	21
Below the mean for geometry	6	14	58	52	36	23	29	50	54	.	.

School Outliers according to algebra

NAME	COL1	COL2	COL3	COL4	COL5	COL6	COL7	COL8	COL9	COL10	COL11
Above the mean for algebra	30	64	48	1	17	27	31	28	24	7	.
Below the mean for algebra	58	42	40	9	41	16	15	14	11	54	36

School Outliers according to difference between geometry and algebra

NAME	COL1	COL2	COL3	COL4	COL5	COL6
Large differences between rankings on geometry and algebra	9	11	34	14	50	29

3.6 Model 4

Model 4 (as Model 2 but with no random effect of gender at school level)

The model for the fixed part is:

$$\text{predicted geometry score} = 6.96 + 0.46YR8prf + 0.46girl + 0.05s\%A\_C,$$
$$\text{predicted algebra score} = 8.90 + 0.46YR8prf + 0.47girl + 1.26tHE + 1.19 \times stext$$

similar to Model 2. Standard errors are as in the table below:

parameter	estimate	s. error(u)
alg_cons	8.901	0.4397
alg_girl	0.4729	0.2102
alg_YR8pr	0.4635	0.02475
alg_tHE	1.265	0.4965
alg_stext	1.186	0.464
geo_cons	6.956	0.6665
geo_girl	0.3136	0.2028
geo_YR8prf	0.4617	0.03036
geo_s%A_C	0.05239	0.01184

The estimated residual variance/correlation matrix at school level is:

$$\begin{array}{c|cc} & G & A \\ \hline G & 1.675 & \\ A & r = .243 & 2.602 \end{array}.$$

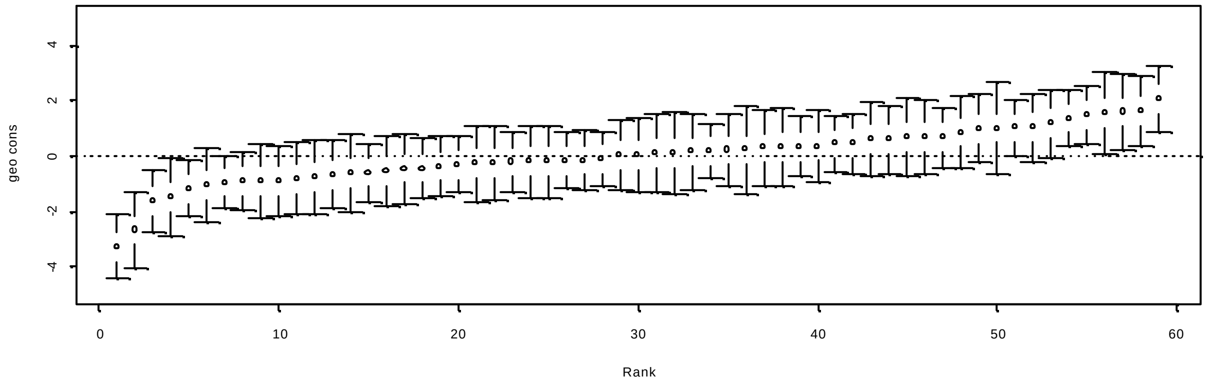
Student level (variances on the diagonal; correlations elsewhere)

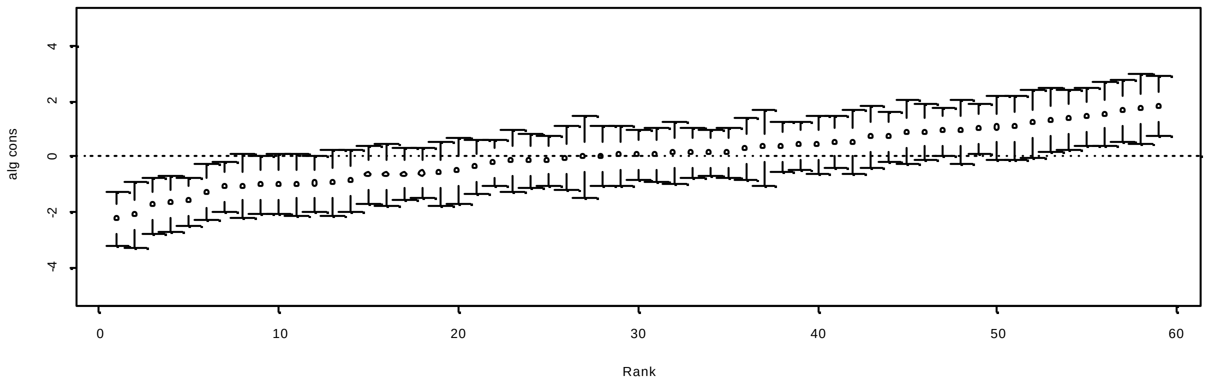
		Girls		*Boys*	
		Geo	*Alg*	*Geo*	*Alg*
Girls	*Geo*	18.64			
	Alg	$r = .29$	19.56		
Boys	*Geo*			19.24	
	Alg			.29	20.29

Compared to Model 2, it is relatively straightforward to detect correlation at school level between performance in Geometry and in Algebra. Statistical test shows that there is no difference in variance between boys' Algebra and girls'. The full tabulation of the random part is:

PARAMETER	ESTIMATE	S. ERROR	CORR.
School			
$\sigma_v^2(\text{geo\_cons})$	1.675	0.4239	1
$\sigma_v(\text{geo\_cons, alg\_cons})$	0.5078	0.3692	0.243
$\sigma_v^2(\text{alg\_cons})$	2.602	0.6043	1
Student			
$\sigma_u^2(\text{geo\_girl})$	18.64	0.8549	1

$\sigma_u(\text{alg\_girl, geo\_girl})$	5.514	0.6454	0.289
$\sigma^2_u(\text{alg\_girl})$	19.56	0.8989	1
$\sigma^2_u(\text{geo\_boy})$	19.24	0.8908	1
$\sigma_u(\text{alg\_boy, geo\_boy})$	5.896	0.6848	0.294
$\sigma^2_u(\text{alg\_boy})$	20.29	0.9689	1





School Residual Ranks from Model 4

School	Geometry	Algebra
1	46	54
2	34	30
3	13	15
4	17	29
5	43	43
6	5	17
7	27	44
8	19	31
9	59	3
10	10	20
11	44	4
12	28	39
13	22	23
14	20	5
15	33	8
16	25	11
17	53	59
18	9	16
20	51	33
21	42	25
22	29	9
23	3	21
24	26	47
25	39	41
26	16	40
27	57	50
28	49	46
29	4	56
30	56	52
31	40	57
32	21	26
33	35	36
34	52	12
35	23	35
36	32	2
37	45	19
39	36	37
40	30	14
41	11	10
42	18	6
43	38	45
44	7	22
45	54	38
46	14	13
48	58	55
50	2	53
51	48	28
52	8	18
53	6	48
54	1	1
55	24	32
56	37	42
57	47	24
58	15	7
59	41	34
60	50	27
61	12	51
62	55	49
64	31	58

Schools ranked according to their residuals for geometry (Model 4)

School	Geometry	Algebra
9	59	3
48	58	55
27	57	50
30	56	52
62	55	49
45	54	38
17	53	59
34	52	12
20	51	33
60	50	27
28	49	46
51	48	28
57	47	24
1	46	54
37	45	19
11	44	4
5	43	43
21	42	25
59	41	34
31	40	57
25	39	41
43	38	45
56	37	42
39	36	37
33	35	36
2	34	30
15	33	8
36	32	2
64	31	58
40	30	14
22	29	9
12	28	39
7	27	44
24	26	47
16	25	11
55	24	32
35	23	35
13	22	23
32	21	26
14	20	5
8	19	31
42	18	6
4	17	29
26	16	40
58	15	7
46	14	13
3	13	15
61	12	51
41	11	10
10	10	20
18	9	16
52	8	18
44	7	22
53	6	48
6	5	17
29	4	56
23	3	21
50	2	53
54	1	1

Schools ranked according to their residuals for algebra (Model 4)

School	Geometry	Algebra
17	53	59
64	31	58
31	40	57
29	4	56
48	58	55
1	46	54
50	2	53
30	56	52
61	12	51
27	57	50
62	55	49
53	6	48
24	26	47
28	49	46
43	38	45
7	27	44
5	43	43
56	37	42
25	39	41
26	16	40
12	28	39
45	54	38
39	36	37
33	35	36
35	23	35
59	41	34
20	51	33
55	24	32
8	19	31
2	34	30
4	17	29
51	48	28
60	50	27
32	21	26
21	42	25
57	47	24
13	22	23
44	7	22
23	3	21
10	10	20
37	45	19
52	8	18
6	5	17
18	9	16
3	13	15
40	30	14
46	14	13
34	52	12
16	25	11
41	11	10
22	29	9
15	33	8
58	15	7
42	18	6
14	20	5
11	44	4
9	59	3
36	32	2
54	1	1

School Outliers according to geometry

NAME	COL1	COL2	COL3	COL4	COL5	COL6
Above the mean for geometry	9	48	27	30	62	45
Below the mean for geometry	44	6	29	23	50	54

School Outliers according to algebra

NAME	COL1	COL2	COL3	COL4	COL5	COL6	COL7	COL8	COL9
Above the mean for algebra	17	64	31	29	48	1	50	62	24
Below the mean for algebra	58	42	14	11	9	36	54	.	.

School Outliers according to difference between geometry and algebra

NAME	COL1	COL2	COL3	COL4	COL5
Large differences between rankings on geometry and algebra	9	11	36	34	14

NAME	COL6	COL7	COL8	COL9
Large differences between rankings on geometry and algebra	61	53	29	50

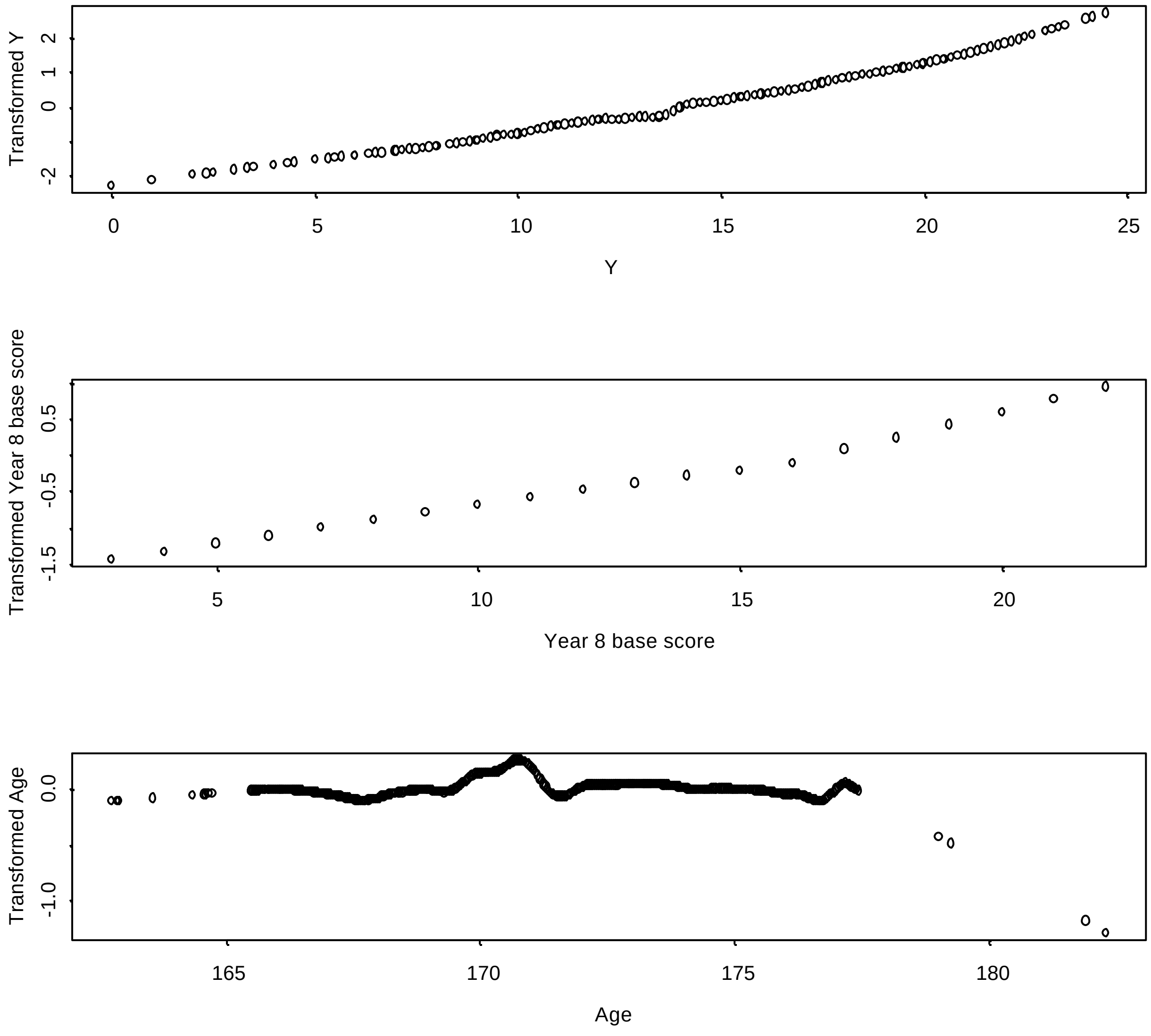
Large changes in rankings from Model 3 to Model 4

School ID	Change in geometry	School ID	Change in algebra
26	-23	2	-16
53	-17	10	-15
41	-16	8	-10
40	16	25	12
11	20	45	12
36	27	12	15

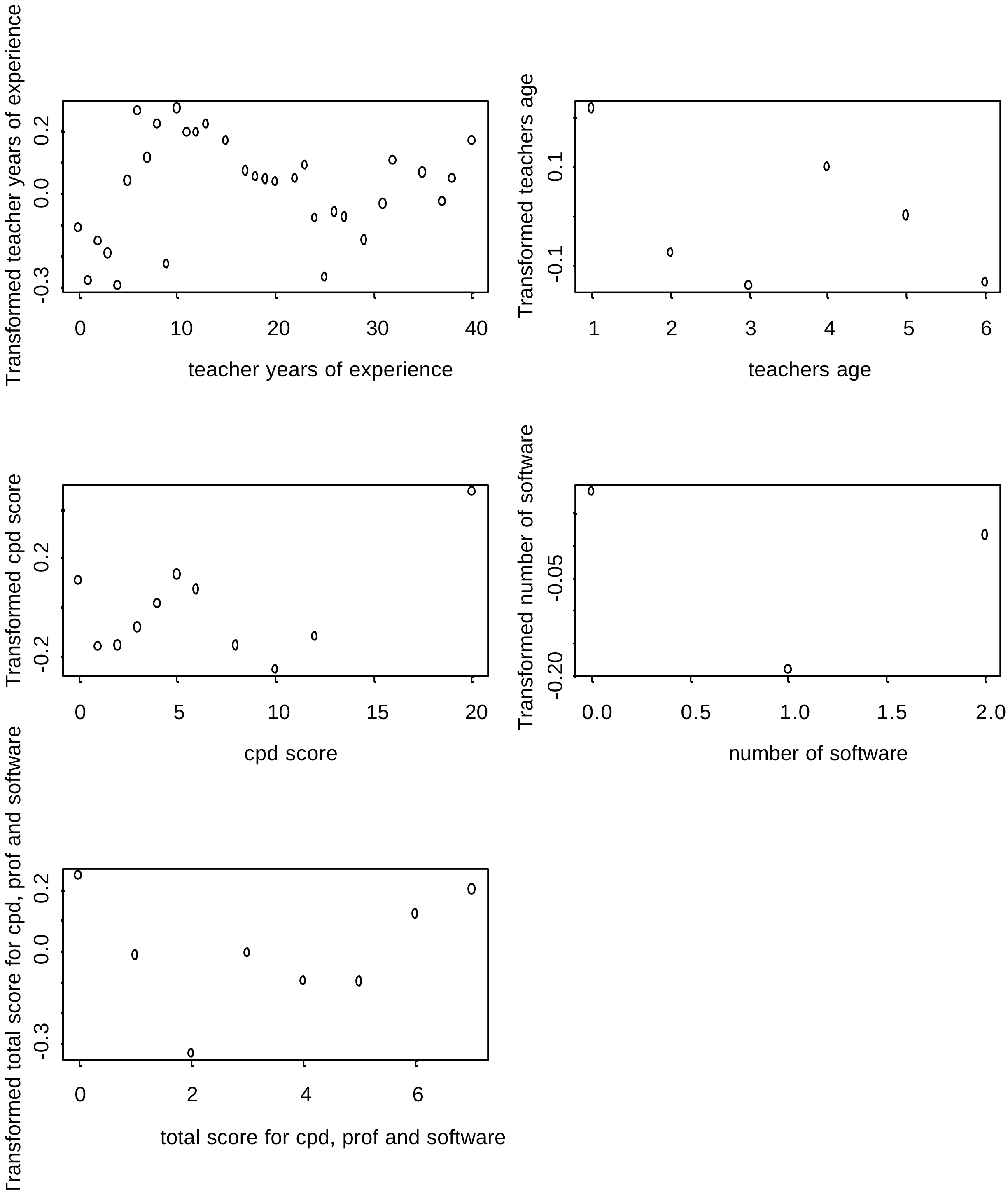
4 Modeling of the Year 9 Proof Score using the Year 8 Baseline Test Score as Baseline

4.1 Variable Codings as suggested by ACE algorithm

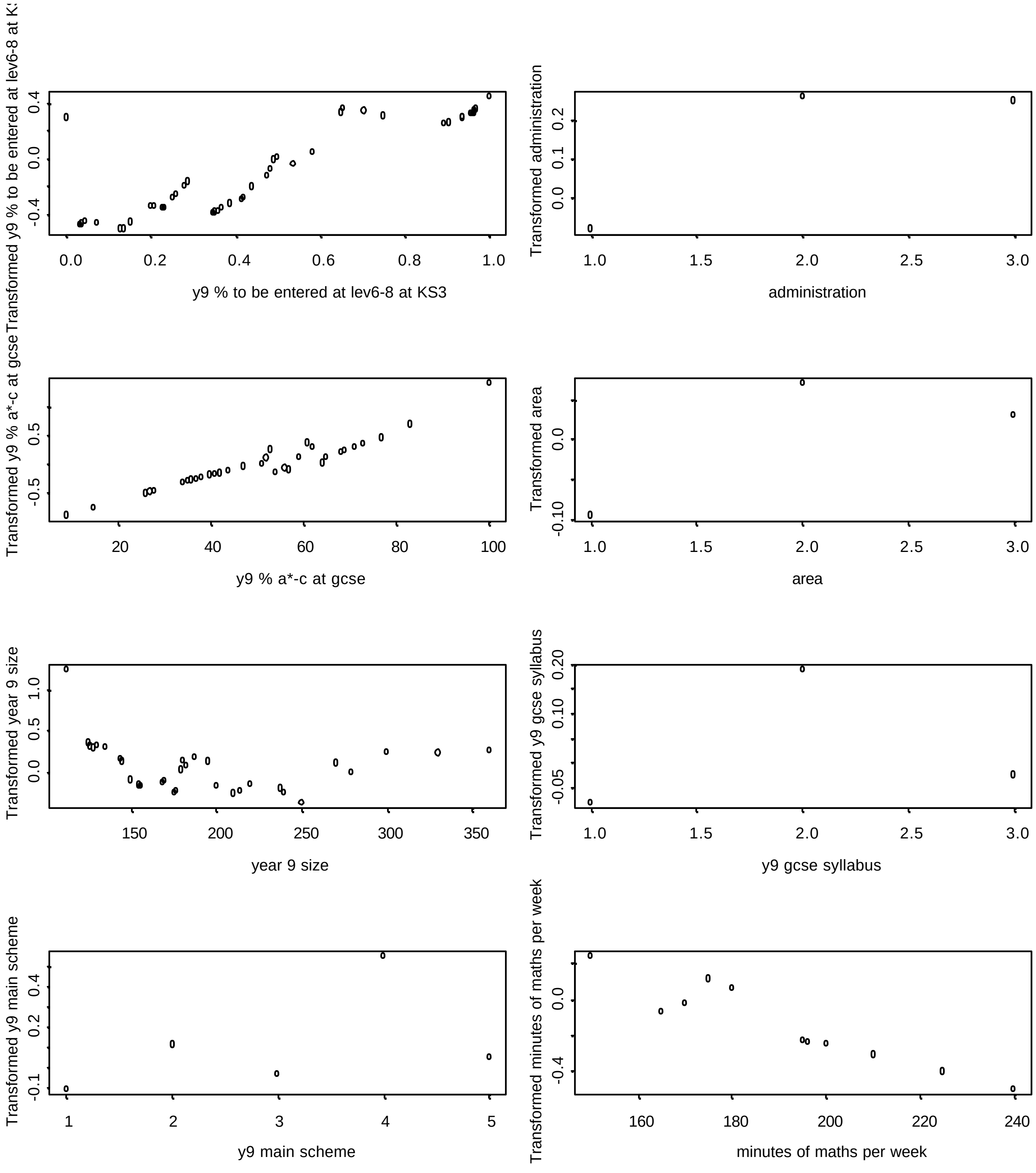
After application of ACE algorithm, the following ACE plots for various variables are produced and displayed in the following fix pages.



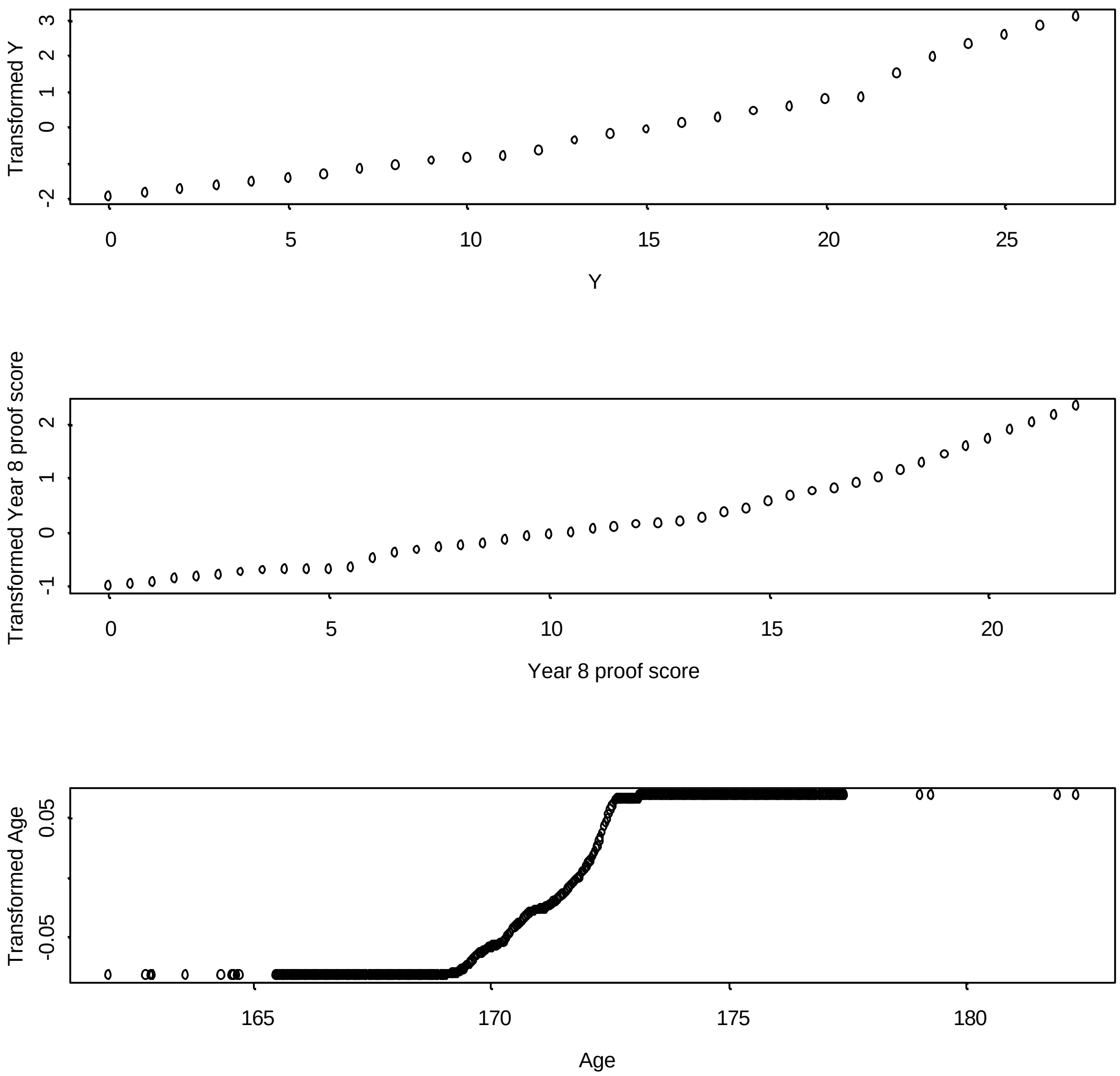
ACE plots for geometry analysis



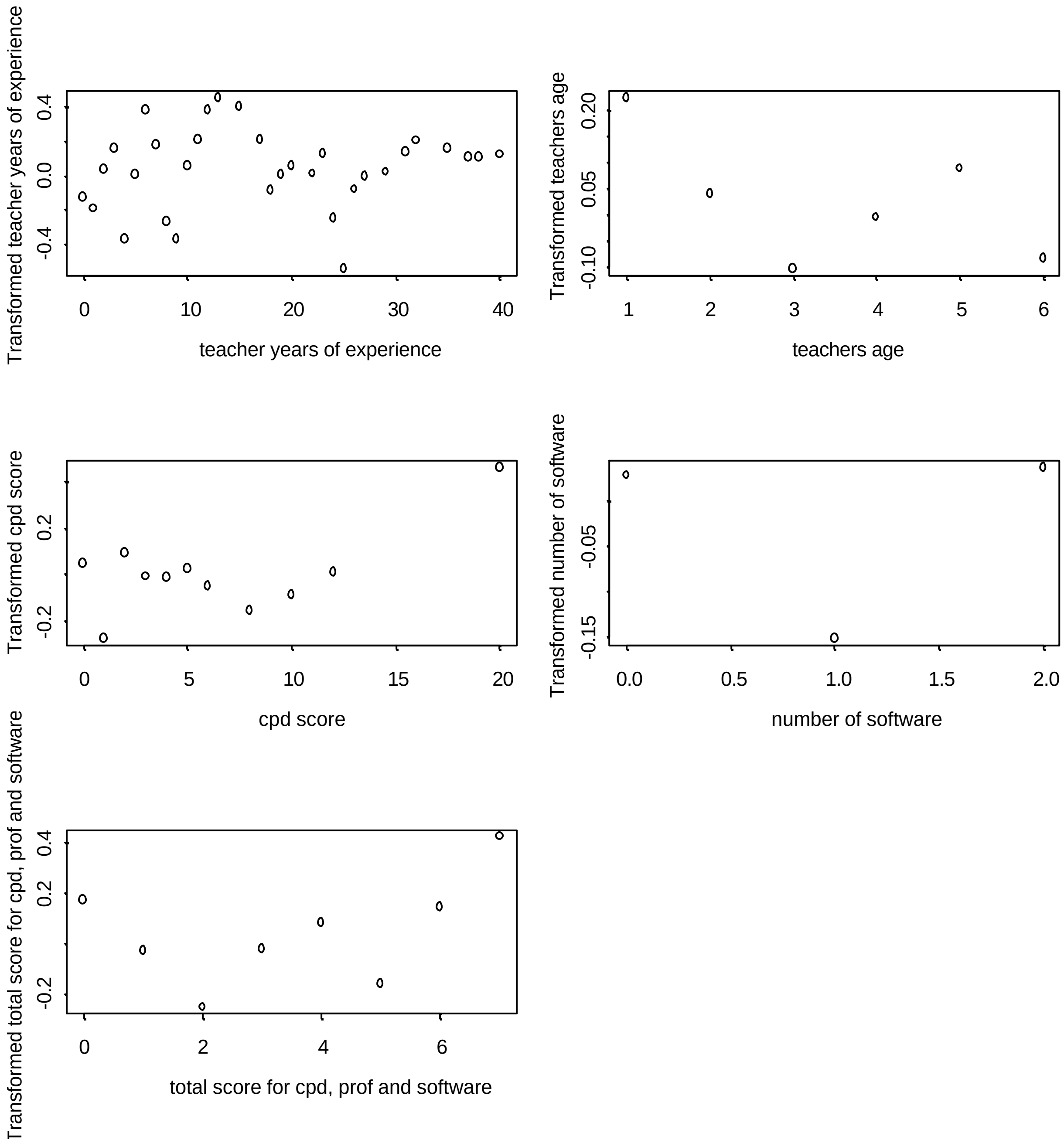
ACE plots for geometry analysis



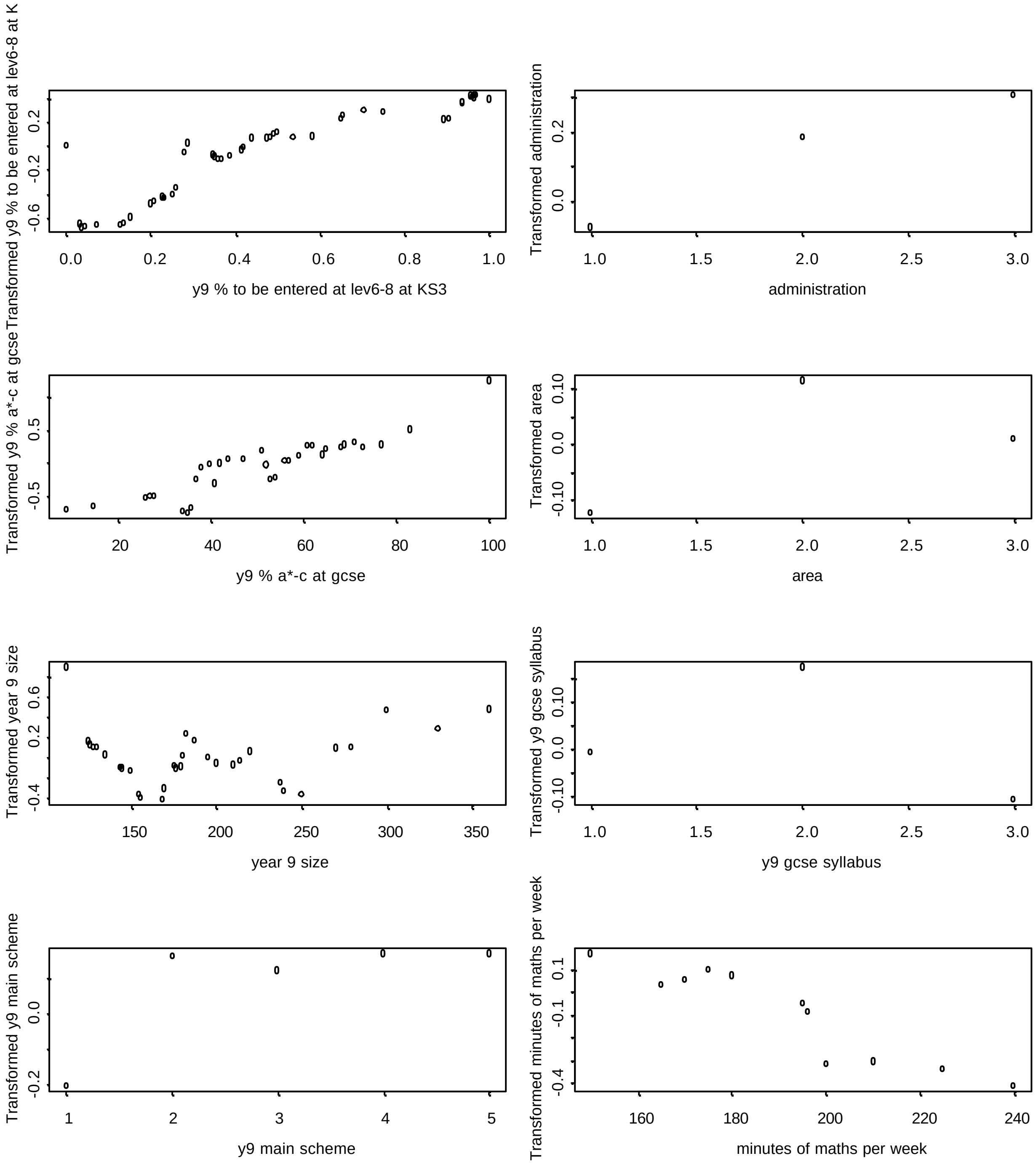
ACE plots for geometry analysis



ACE plots for algebra analysis



ACE plots for algebra analysis



ACE plots for algebra analysis

Based on the ACE plots above and BIC statistic calculation, we decided the following transformation or codings for the variable in the analysis:

Variables	Name	Variable Transformation or Codings
Response Variable		
Year 9 proof score	YR9prf	linear
Predictor Variables		
Year 8 baseline test score	BLMtest	linear score
	BLMtest2	quadratic score
school-level:		
administration	sadmin	0=county, 1=voluntary (VA/VC) or other
age range 11-16 or 11-18	sage	1=11-16, 0=11-18
% A*-C at GCSE	s%A_C	linear
gender	smix	1=girls only, 0=mixed
area (one of three)	sarea	1=urban or suburban, 0=rural
year-9 size	ssize	linear(school size)
% to be entered at lev6-8 at KS3	spent1	min(Year 9 % enter at KS3, 15%)
	spent2	max(0,min(Year 9 % at KS3-15%,25%-15%))"
	spent3	max(0,Year 9 % enter at KS3-25%)
GCSE syllabus	ssyll	1=OCR,0=Others
maths textbook or scheme in use	stext	1=SMP/Vickers/ST(P)/other,0=Key Math
minutes of maths per week	smtime	linear (minutes)
existence of maths club	smclub	1=Yes,0=No
class-level:		
teacher gender	tsex	1=Female, 0=Male
teacher years of experience	tyearex1	min(year,12)
	tyearex2	max(0,min(year-12,22-12)
	tyearex3	max(0,year-22)
teacher age	tage	linear(year)
teacher degree	tdegree	1=having a maths degree, 0=otherwise
teacher PGCE or Cert	tpgrt	1=having a PGCE or Cert,0=otherwise
teacher HE	tHE	1=having MSc or PhD degree, 0=otherwise
teacher CPD score	tcpd	linear(score)
teacher membership of a professional assoc.	tmember	1=Yes, 0=No
teacher knowledge/use of software	tsoft	1=Yes,0=No
teacher total score for CPD, PROF and software	ttscore	linear(score)
student-level:		
gender	girl	1=Girl,0=Boy
age in months	age	linear(month)

4.2 Model 0

The fixed-part estimates from this model may be expressed by means of the two equations:

$$\text{predicted geometry score} = 13.54 + 0.29\,girl,$$

$$\text{predicted algebra score} = 14.35 + 0.40\,girl,$$

thus, the predicted score for the base group (boys) in Geometry is 13.54(s.e. 0.32) and in Algebra it is 14.35 (s.e. 0.36). The gender coefficients are statistically non-significant, as is shown in the full tabulation below:

parameter	estimate	s. error(u)
alg_cons	14.35	0.3627
alg_girl	0.4039	0.2375
geo_cons	13.54	0.322
geo_girl	0.2884	0.2219

Thus, there is *no* statistically significant gender effect on either outcome score when the model makes no adjustment for baseline score.

The residual variance/covariance matrices for the outcome scores at the three levels, school, class, and student, have the following estimates (covariances have been converted to correlations for convenience):

School level

	Geo	*Alg*
Geo	2.98	
Alg	$r = .84$	4.27

Class level

	Geo	*Alg*
Geo	.2.32	
Alg	$r = .72$	2.47

Student level

	Geo	*Alg*
Geo	20.86	
Alg	$r = .41$	23.85

The full tabulation is:

PARAMETER	ESTIMATE	S. ERROR	CORR.
School			
σ^2_w(geo_cons)	2.983	1.131	1
σ_w(geo_cons, alg_cons)	3.013	1.108	0.844
σ^2_w(alg_cons)	4.275	1.415	1
Class			
σ^2_v(geo_cons)	2.321	0.8567	1
σ_v(geo_cons, alg_cons)	1.731	0.7512	0.723
σ^2_v(alg_cons)	2.471	0.9442	1
Student			
σ^2_u(geo_cons)	20.86	0.6973	1
σ_u(geo_cons, alg_cons)	9.179	0.5703	0.412
σ^2_u(alg_cons)	23.85	0.7974	1

Two important observations can be found from the above results. First, there is statistically significant, though small, residual variation at class level, and high residual correlation ($r \approx .84$) at both school and class level between the two outcomes. Thus, in this very simple model, schools that perform above the average in algebra are predicted to do so in geometry also, and vice versa.

Second, class effects on the two subjects, within schools, are predicted to be similar. A student, however, who performs above the expectation for her class in algebra has only a slight tendency to perform above expectation in geometry also. In percentages, the residual variances at school, class, and student levels, are in the ratio 2.98:2.32:20.86 for Geometry and 4.27:2.47:23.85 for Algebra.

4.3 Model 1

We have included the baseline test score BLMtest in this model as a predictor. As suggested by the ACE algorithm, the relationship between year 9 proof score and BLMtest score can be best described by a quadratic function as follows for geometry scores:

$$\text{predicted geometry score} = 9.43 - 0.25 BLMtest + 0.03 BLMtest^2 + 1.20 girl ,$$

The corresponding model for algebra scores is

$$\text{predicted algebra score} = 6.41 + 0.19 BLMtest + 0.01 BLMtest^2 + 1.03 girl ,$$

The full tabulation of the fixed part, showing the standard errors, is:

```
parameter            estimate     s. error(u)
alg_cons                 9.43        1.581
alg_girl                1.396        0.3509
alg_BLMtest           -0.2529        0.2093
alg_BLMtest2          0.03186      0.006877

geo_cons                 6.41        1.503
geo_girl                1.028        0.2438
geo_BLMtest            0.1943        0.1981
geo_BLMtest2          0.01397      0.006498
```

Once baseline score is included, no statistically significant variation remains at class level within school. Thus, we have two levels of variation, students at level 2 within schools at level 3, with level 1 used as before to distinguish between the responses in Algebra and Geometry.

The relationship described by each equation is illustrated in the first of the two graphs on the following pages. The second graph shows the relationship of the outcome scores with the raw baseline score.

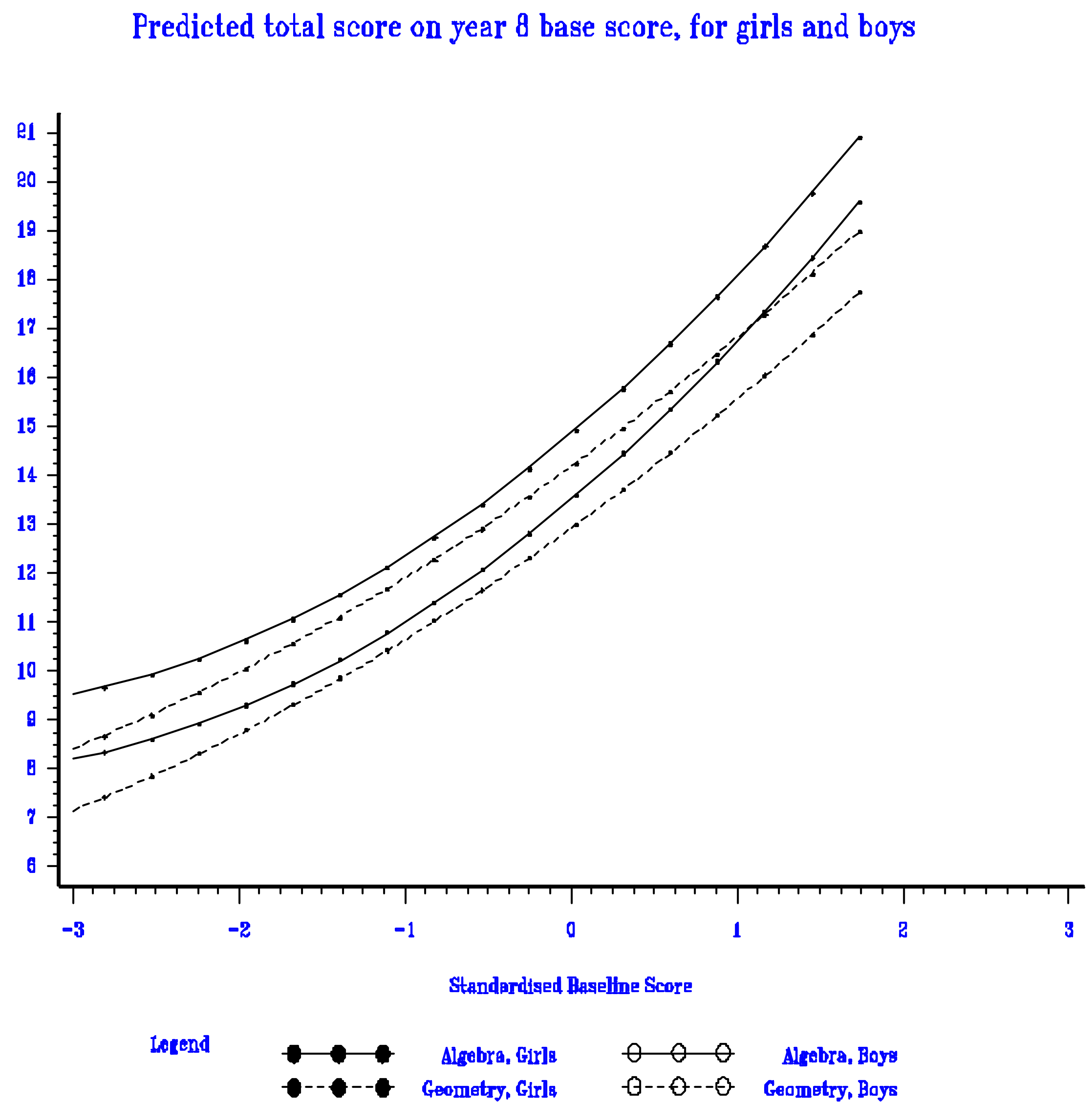
The random part of Model 1 can be expressed as follows:

School level (variances on the diagonal; correlations elsewhere)

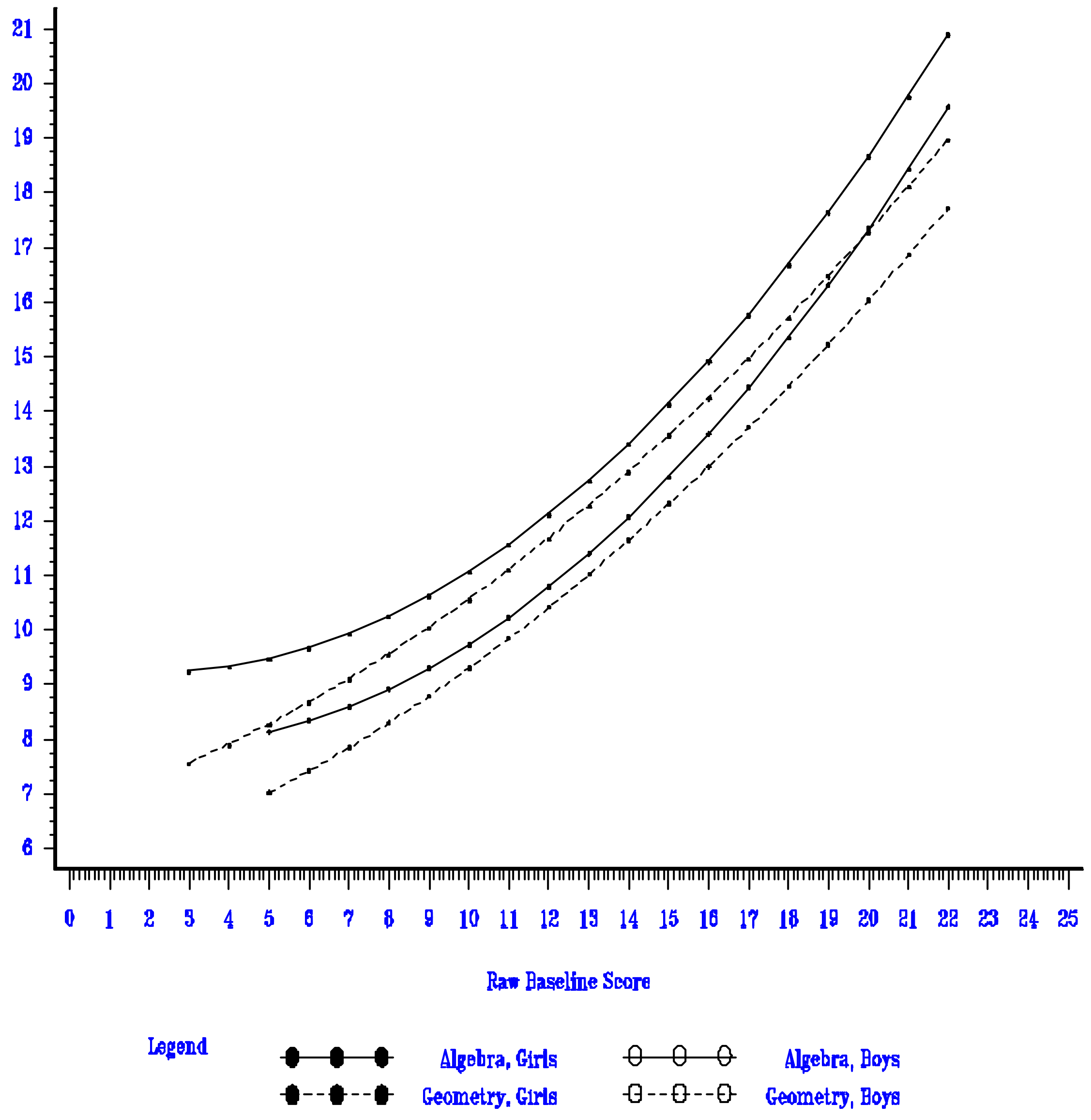
		Girls		*Boys*	
		Geo	*Alg*	*Geo*	*Alg*
Girls	*Geo*	1.752			
	Alg		2.166		
Boys	*Geo*	$r = .81$		2.486	
	Alg				1.992

Student level (variances on the diagonal; correlations elsewhere)

		Girls		*Boys*	
		Geo	*Alg*	*Geo*	*Alg*
Girls	*Geo*	17.83			
	Alg	$r = .29$	19.15		
Boys	*Geo*			18.61	
	Alg			.357	22.14


Predicted total score on year 8 base score, for girls and boys
21
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
-3
-2
-1
0
1
2
3
Standardised Baseline Score
Legend
Algebra, Girls
Geometry, Girls
Algebra, Boys
Geometry, Boys

Predicted total score on year 8 base score, for girls and boys

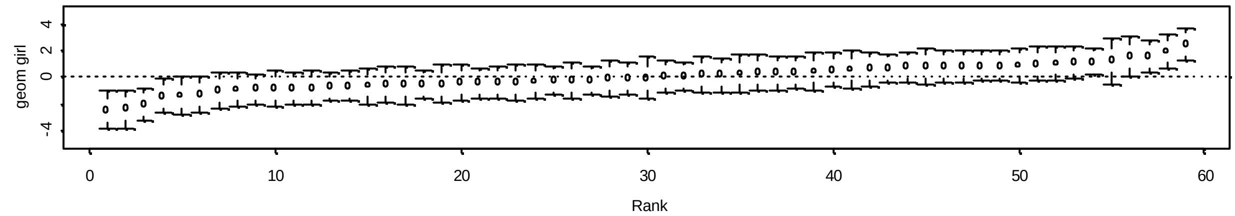


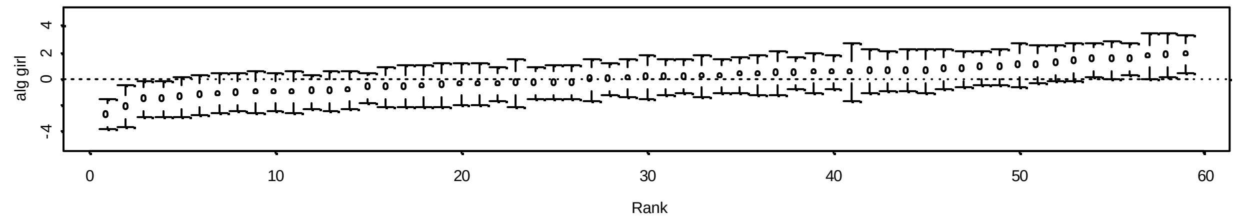
The full tabulation is:

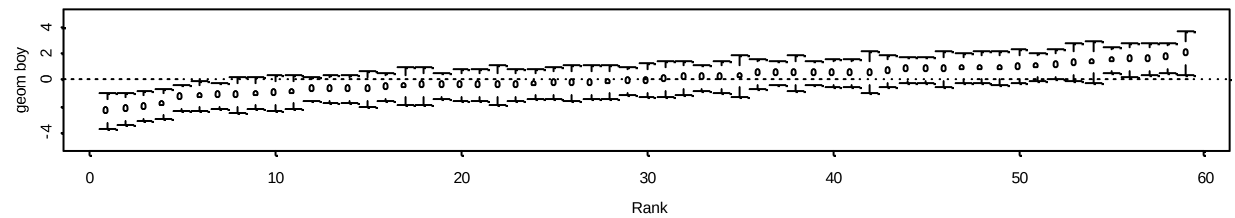
PARAMETER	ESTIMATE	S. ERROR	CORR.
School			
$\sigma_v^2(\text{geo\_girl})$	1.752	0.5312	1
$\sigma_v^2(\text{alg\_girl})$	2.166	0.6259	1
$\sigma_v(\text{geo\_boy, geo\_girl})$	1.696	0.4869	0.81
$\sigma_v^2(\text{geo\_boy})$	2.486	0.6891	1
$\sigma_v^2(\text{alg\_boy})$	1.992	0.637	1
Student			
$\sigma_u^2(\text{geo\_girl})$	17.8	0.8411	1
$\sigma_u(\text{alg\_girl, geo\_girl})$	5.451	0.6405	0.292
$\sigma_u^2(\text{alg\_girl})$	19.15	0.9056	1
$\sigma_u^2(\text{geo\_boy})$	18.61	0.8885	1
$\sigma_u^2(\text{alg\_boy})$	22.14	1.057	1
$s_u(\text{alg\_boy, geo\_boy})$	7.248	0.7244	0.352

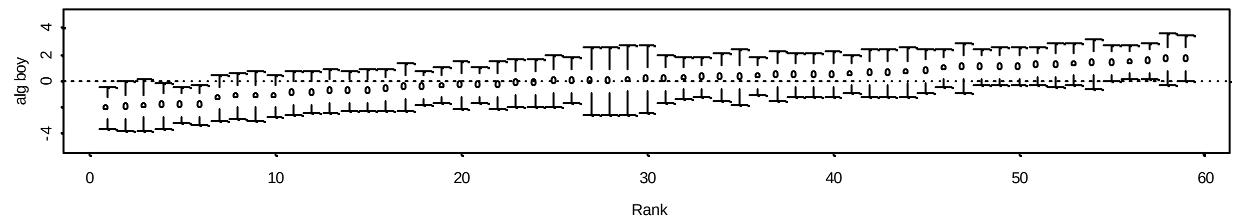
The very high value of correlation between girls' and boys' geometry scores (within subject) at school level means that, for example, a school that has a high residual for boys is very likely to have a high residual also for girls in the same subject.

The following chart plots school-level residuals against their ranks, with error bars corresponding to 1.96 SD. Thus, an error bar wholly above the dotted line corresponds to a school that is performing above the mean predicted by the model, with 95% confidence.









Tables of the school residual ranks now follow (A high-numbered rank indicates good performance.)

School Residual Ranks from Model 1

School	geo_girl	alg_girl	alg_boy	geo_boy
1	57	56	.	.
2	44	52	51	38
3	12	10	11	6
4	6	27	41	9
5	42	48	.	.
6	9	14	5	7
7	31	47	53	33
8	22	29	47	14
9	58	6	15	55
10	8	36	24	23
11	29	3	1	21
12	32	28	46	29
13	23	43	4	28
14	18	1	10	13
15	16	20	9	30
16	26	8	2	26
17	50	51	39	47
18	7	11	17	11
20	54	15	49	45
21	46	35	42	49
22	24	18	13	24
23	3	4	12	3
24	38	59	52	42
25	49	25	29	40
26	33	50	28	35
27	56	57	27	53
28	51	55	35	46
29	2	30	48	2
30	59	54	.	.
31	37	34	.	.
32	10	7	43	10
33	40	16	23	32
34	52	33	16	50
35	14	40	21	5
36	5	2	3	8
37	36	17	20	37
39	30	23	31	22
40	20	9	22	15
41	34	13	7	20
42	21	12	8	25
43	41	58	36	48
44	27	26	18	12
45	43	22	19	36
46	19	21	25	18
48	53	53	55	54
50	1	37	38	1
51	45	39	26	41
52	11	49	32	19
53	35	42	50	31
54	4	5	6	4
55	28	19	34	34
56	39	44	40	44
57	47	31	37	39
58	13	24	14	16
59	25	32	44	43
60	55	41	33	51
61	15	46	30	27
62	48	38	45	52
64	17	45	54	17

Schools ranked according to their residuals for girls geometry (Model 1)

School	geo_girl	alg_girl	alg_boy	geo_boy
30	59	54	.	.
9	58	6	15	55
1	57	56	.	.
27	56	57	27	53
60	55	41	33	51
20	54	15	49	45
48	53	53	55	54
34	52	33	16	50
28	51	55	35	46
17	50	51	39	47
25	49	25	29	40
62	48	38	45	52
57	47	31	37	39
21	46	35	42	49
51	45	39	26	41
2	44	52	51	38
45	43	22	19	36
5	42	48	.	.
43	41	58	36	48
33	40	16	23	32
56	39	44	40	44
24	38	59	52	42
31	37	34	.	.
37	36	17	20	37
53	35	42	50	31
41	34	13	7	20
26	33	50	28	35
12	32	28	46	29
7	31	47	53	33
39	30	23	31	22
11	29	3	1	21
55	28	19	34	34
44	27	26	18	12
16	26	8	2	26
59	25	32	44	43
22	24	18	13	24
13	23	43	4	28
8	22	29	47	14
42	21	12	8	25
40	20	9	22	15
46	19	21	25	18
14	18	1	10	13
64	17	45	54	17
15	16	20	9	30
61	15	46	30	27
35	14	40	21	5
58	13	24	14	16
3	12	10	11	6
52	11	49	32	19
32	10	7	43	10
6	9	14	5	7
10	8	36	24	23
18	7	11	17	11
4	6	27	41	9
36	5	2	3	8
54	4	5	6	4
23	3	4	12	3
29	2	30	48	2
50	1	37	38	1

Schools ranked according to their residuals for girls algebra (Model 1)

School	geo_girl	alg_girl	alg_boy	geo_boy
24	38	59	52	42
43	41	58	36	48
27	56	57	27	53
1	57	56	.	.
28	51	55	35	46
30	59	54	.	.
48	53	53	55	54
2	44	52	51	38
17	50	51	39	47
26	33	50	28	35
52	11	49	32	19
5	42	48	.	.
7	31	47	53	33
61	15	46	30	27
64	17	45	54	17
56	39	44	40	44
13	23	43	4	28
53	35	42	50	31
60	55	41	33	51
35	14	40	21	5
51	45	39	26	41
62	48	38	45	52
50	1	37	38	1
10	8	36	24	23
21	46	35	42	49
31	37	34	.	.
34	52	33	16	50
59	25	32	44	43
57	47	31	37	39
29	2	30	48	2
8	22	29	47	14
12	32	28	46	29
4	6	27	41	9
44	27	26	18	12
25	49	25	29	40
58	13	24	14	16
39	30	23	31	22
45	43	22	19	36
46	19	21	25	18
15	16	20	9	30
55	28	19	34	34
22	24	18	13	24
37	36	17	20	37
33	40	16	23	32
20	54	15	49	45
6	9	14	5	7
41	34	13	7	20
42	21	12	8	25
18	7	11	17	11
3	12	10	11	6
40	20	9	22	15
16	26	8	2	26
32	10	7	43	10
9	58	6	15	55
54	4	5	6	4
23	3	4	12	3
11	29	3	1	21
36	5	2	3	8
14	18	1	10	13

School Outliers according to geometry

NAME	COL1	COL2	COL3	COL4	COL5	COL6	COL7
Above the mean for girls geometry	30	9	1	27	20	.	.
Above the mean for boys geometry	9	48	27	62	30	.	.
Below the mean for girls geometry	54	23	29	50	.	.	.
Below the mean for boys geometry	6	3	35	54	23	29	50

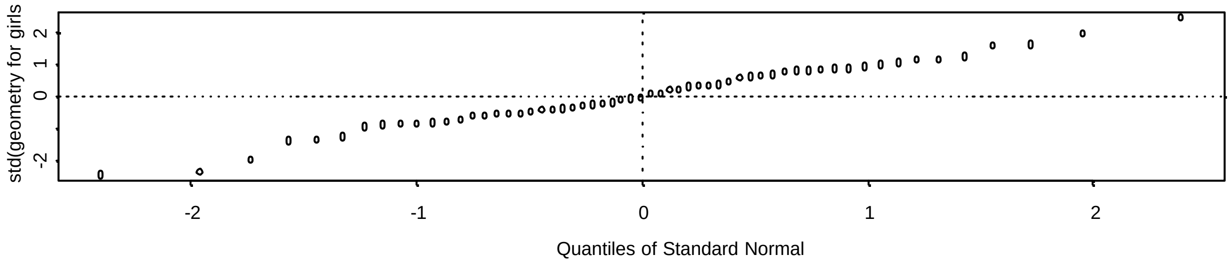
School Outliers according to algebra

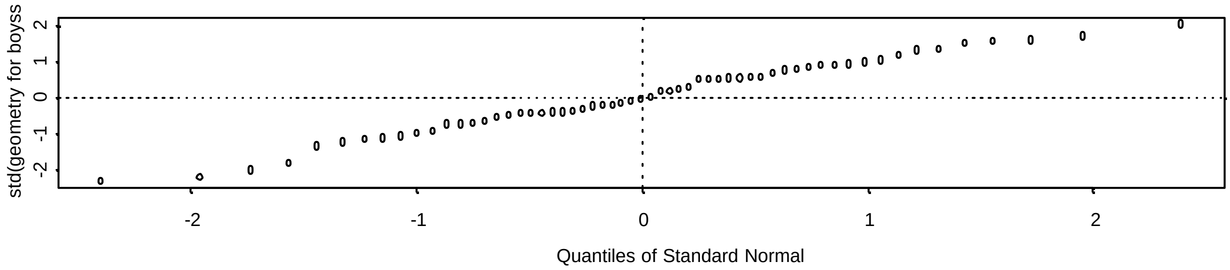
NAME	COL1	COL2	COL3	COL4	COL5	COL6
Above the mean for girls algebra	24	43	27	1	28	30
Above the mean for boys algebra	7	24	2	.	.	.
Below the mean for girls algebra	23	11	36	14	.	.
Below the mean for boys algebra	54	6	13	16	11	.

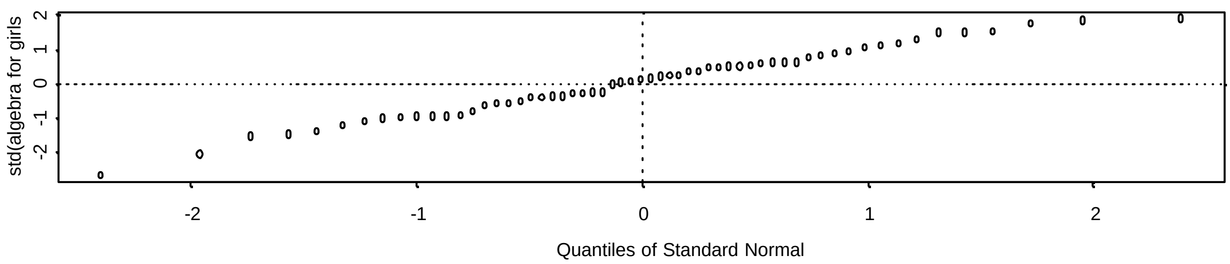
School Outliers according to difference between geometry and algebra

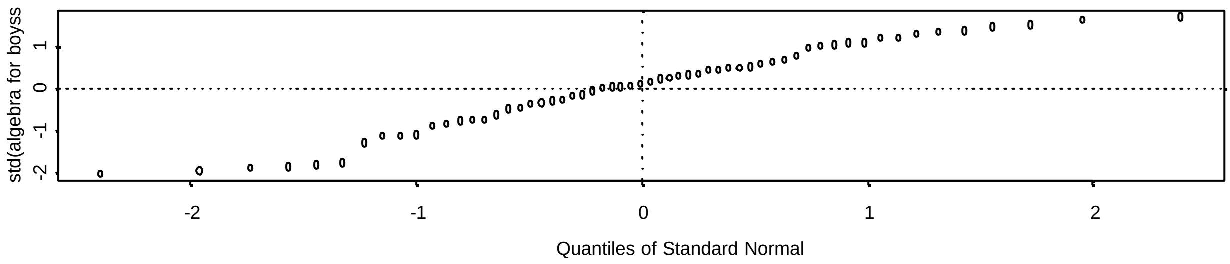
NAME	COL1	COL2	COL3	COL4	COL5
Large differences between rankings on girls geometry and algebra	9	14	20	29	50
Large differences between rankings on boys geometry and algebra	29	50	9	8	.

We also plotted standardised diagnostic school-level residuals against their normal scores (see below). These are not ideal, and reflect problems in the scoring of the outcome.


std(geometry for girls
Quantiles of Standard Normal


std(geometry for boyss
Quantiles of Standard Normal


std(algebra for girls
Quantiles of Standard Normal


std(algebra for boyss
Quantiles of Standard Normal

4.4 Model 2

Model 2 is the most parsimonious model selected by forward and backward procedure.

The fixed part of Model 2 is:

$$\text{predicted geometry score} = 4.10 + 0.21\text{BLMtest} + 0.01\text{BLMtest}^2 + 1.00 girl + 0.05 s\%A\_C,$$

$$\text{predicted algebra score} = 7.90 - 0.25\text{BLMtest} + 0.03\text{BLMtest}^2 + 1.41 girl + 0.24 tage + 1.16 tHE + 1.13 stext$$

parameter	estimate	s. error(u)
alg_cons	7.9	1.64
alg_girl	1.412	0.342
alg_BLMtest	-0.2524	0.2083
alg_BLMtest2	0.03145	0.006848
alg_tage	0.2366	0.1203
alg_tHE	1.156	0.4798
alg_stext	1.127	0.3424
geo_cons	4.109	1.58
geo_girl	1.003	0.2435
geo_BLMtest	0.2071	0.1969
geo_BLMtest2	0.01277	0.006465
geo_s%A_C	0.04672	0.0111

The results show that s%A_C (the school % GCSE pass rate at A*-C) statistically significantly associated with the geometry score: 10% increase in this variable will result in .5 proof score.

Three additional variables are found to be significant predictors of proof algebra score. A teacher with a MSc or PhD degree education will on average increase the proof score by 1.16 and students using of textbooks other than “Key Math“ will have 1.13 higher proof score than those using other textbooks. In addition, the older the teacher the higher the proof score.

Turning to the random part of Model 2, we find that including the additional predictor in the fixed part reduces the school-level variation. Schools’ residual performance in algebra is significantly more variable for girls than for boys. There is little change from Model 1 in the random part at student level.

School level (variances on the diagonal; correlations elsewhere)

		Girls		*Boys*	
		Geo	*Alg*	*Geo*	*Alg*
Girls	*Geo*	1.053			
	Alg	$r = .83$	1.91		
Boys	*Geo*			2.264	
	Alg				1.91

Student level (variances on the diagonal; correlations elsewhere)

		Girls		*Boys*	
		Geo	*Alg*	*Geo*	*Alg*
Girls	*Geo*	17.84			
	Alg	$r = .29$	19.01		
Boys	*Geo*			18.55	
	Alg			.36	22.02

The full tabulation of the random part of Model 2 is:

PARAMETER	ESTIMATE	S. ERROR	CORR.
School			
σ_v^2(geo_girl)	1.053	0.3973	1
σ_v^2(alg_girl)	1.91	0.574	1
σ_v(geo_boy, geo_girl)	1.236	0.3989	0.833
s_v(geo_boy)	2.264	0.6471	1
σ_v^2(alg_boy)	1.91	0.6199	1
Student			
σ_u^2(geo_girl)	17.84	0.8412	1
σ_u(alg_girl, geo_girl)	5.39	0.6366	0.293
σ_u^2(alg_girl)	19.01	0.8987	1
σ_u^2(geo_boy)	18.55	0.8857	1
σ_u(alg_boy, geo_boy)	7.199	0.721	0.356
σ_u^2(alg_boy)	22.02	1.051	1

School Residual Ranks from Model 2

School	geo_girl	alg_girl	alg_boy	geo_boy
1	42	55	.	.
2	48	43	41	36
3	5	11	15	4
4	9	28	34	11
5	49	52	.	.
6	6	17	5	6
7	21	41	50	21
8	19	25	36	14
9	59	2	9	55
10	7	22	8	15
11	47	6	6	38
12	41	37	52	31
13	11	40	1	17
14	33	1	17	26
15	27	29	12	40
16	39	10	3	37
17	45	57	47	44
18	8	9	18	8
20	51	7	44	45
21	40	35	42	41
22	31	15	14	29
23	2	4	13	3
24	35	58	45	39
25	38	33	43	27
26	14	47	25	22
27	56	54	19	50
28	44	53	31	43
29	4	48	53	5
30	55	36	.	.
31	25	42	.	.
32	22	12	49	16
33	29	21	32	20
34	52	23	11	51
35	20	38	21	7
36	26	3	4	33
37	50	16	22	52
39	34	32	40	19
40	37	13	26	28
41	15	20	16	9
42	28	8	7	32
43	30	59	33	35
44	16	34	28	12
45	46	27	29	42
46	24	19	23	25
48	43	49	51	48
50	1	56	46	1
51	53	31	20	47
52	13	46	30	23
53	18	50	55	13
54	3	5	2	2
55	23	14	27	30
56	32	39	38	34
57	58	24	39	46
58	17	18	10	24
59	36	26	37	49
60	57	30	24	53
61	12	51	35	18
62	54	45	48	54
64	10	44	54	10

Schools ranked according to their residuals for girls geometry (Model 2)

School	geo_girl	alg_girl	alg_boy	geo_boy
9	59	2	9	55
57	58	24	39	46
60	57	30	24	53
27	56	54	19	50
30	55	36	.	.
62	54	45	48	54
51	53	31	20	47
34	52	23	11	51
20	51	7	44	45
37	50	16	22	52
5	49	52	.	.
2	48	43	41	36
11	47	6	6	38
45	46	27	29	42
17	45	57	47	44
28	44	53	31	43
48	43	49	51	48
1	42	55	.	.
12	41	37	52	31
21	40	35	42	41
16	39	10	3	37
25	38	33	43	27
40	37	13	26	28
59	36	26	37	49
24	35	58	45	39
39	34	32	40	19
14	33	1	17	26
56	32	39	38	34
22	31	15	14	29
43	30	59	33	35
33	29	21	32	20
42	28	8	7	32
15	27	29	12	40
36	26	3	4	33
31	25	42	.	.
46	24	19	23	25
55	23	14	27	30
32	22	12	49	16
7	21	41	50	21
35	20	38	21	7
8	19	25	36	14
53	18	50	55	13
58	17	18	10	24
44	16	34	28	12
41	15	20	16	9
26	14	47	25	22
52	13	46	30	23
61	12	51	35	18
13	11	40	1	17
64	10	44	54	10
4	9	28	34	11
18	8	9	18	8
10	7	22	8	15
6	6	17	5	6
3	5	11	15	4
29	4	48	53	5
54	3	5	2	2
23	2	4	13	3
50	1	56	46	1

Schools ranked according to their residuals for girls algebra (Model 2)

School	geo_girl	alg_girl	alg_boy	geo_boy
43	30	59	33	35
24	35	58	45	39
17	45	57	47	44
50	1	56	46	1
1	42	55	.	.
27	56	54	19	50
28	44	53	31	43
5	49	52	.	.
61	12	51	35	18
53	18	50	55	13
48	43	49	51	48
29	4	48	53	5
26	14	47	25	22
52	13	46	30	23
62	54	45	48	54
64	10	44	54	10
2	48	43	41	36
31	25	42	.	.
7	21	41	50	21
13	11	40	1	17
56	32	39	38	34
35	20	38	21	7
12	41	37	52	31
30	55	36	.	.
21	40	35	42	41
44	16	34	28	12
25	38	33	43	27
39	34	32	40	19
51	53	31	20	47
60	57	30	24	53
15	27	29	12	40
4	9	28	34	11
45	46	27	29	42
59	36	26	37	49
8	19	25	36	14
57	58	24	39	46
34	52	23	11	51
10	7	22	8	15
33	29	21	32	20
41	15	20	16	9
46	24	19	23	25
58	17	18	10	24
6	6	17	5	6
37	50	16	22	52
22	31	15	14	29
55	23	14	27	30
40	37	13	26	28
32	22	12	49	16
3	5	11	15	4
16	39	10	3	37
18	8	9	18	8
42	28	8	7	32
20	51	7	44	45
11	47	6	6	38
54	3	5	2	2
23	2	4	13	3
36	26	3	4	33
9	59	2	9	55
14	33	1	17	26

School Outliers according to geometry

NAME	COL1	COL2	COL3	COL4	COL5	COL6
Above the mean for girls geometry	9	.	.	.	.	.
Above the mean for boys geometry	9	62	34	59	.	.
Below the mean for girls geometry	54	23	50	.	.	.
Below the mean for boys geometry	6	29	3	23	54	50

School Outliers according to algebra

NAME	COL1	COL2	COL3	COL4	COL5
Above the mean for girls algebra	24	17	.	.	.
Above the mean for boys algebra	.	.	.	.	.
Below the mean for girls algebra	20	23	36	9	14
Below the mean for boys algebra	6	54	13	.	.

School Outliers according to difference between geometry and algebra

NAME	COL1	COL2	COL3	COL4	COL5	COL6
Large differences between rankings on girls geometry and algebra	9	14	20	11	50	.
Large differences between rankings on boys geometry and algebra	50	34	29	64	9	53

Largest changes in rankings from model 1 to model 2

School ID	Change in geo_girl	School ID	Change in alg_girl	School ID	Change in geo_boy	School ID	Change in alg_boy
26	-19	30	-18	53	-18	10	-16
41	-19	10	-14	26	-13	8	-11
53	-17	60	-11	43	-13	2	-10
40	17	15	9	37	15	44	10
11	18	29	18	11	17	45	10
36	21	50	19	36	25	25	14

The next two models are included to illustrate what happens when the random effect at school level is removed from gender and attached instead to the intercept term, which is common to both boys and girls. This is equivalent to pooling schools' performances for their boys and their girls, in other words assuming their 'effects' are the same for either gender. The fixed part changes very little, but a slightly different ranking of schools arises.

4.5 Model 3

Model 3 (as Model 1, but with no random effect of gender at school level)

The model for the fixed part is:

$$\text{predicted geometry score} = 6.92 + 0.14\text{BLMtest} + 0.01\text{BLMtest}^2 + 1.02 girl,$$

$$\text{predicted algebra score} = 10.21 - 0.31\text{BLMtest} + 0.03\text{BLMtest}^2 + 1.24 girl,$$

parameter	estimate	s. error(u)
alg_cons	10.21	1.58
alg_girl	1.239	0.2194
alg_BLMtest	-0.3131	0.2085
alg_BLMtest2	0.03293	0.006846
geo_cons	6.917	1.507
geo_girl	1.021	0.2076
geo_BLMtest	0.1415	0.1987
geo_BLMtest2	0.01527	0.006515

The estimated residual variance/correlation matrix at school level is

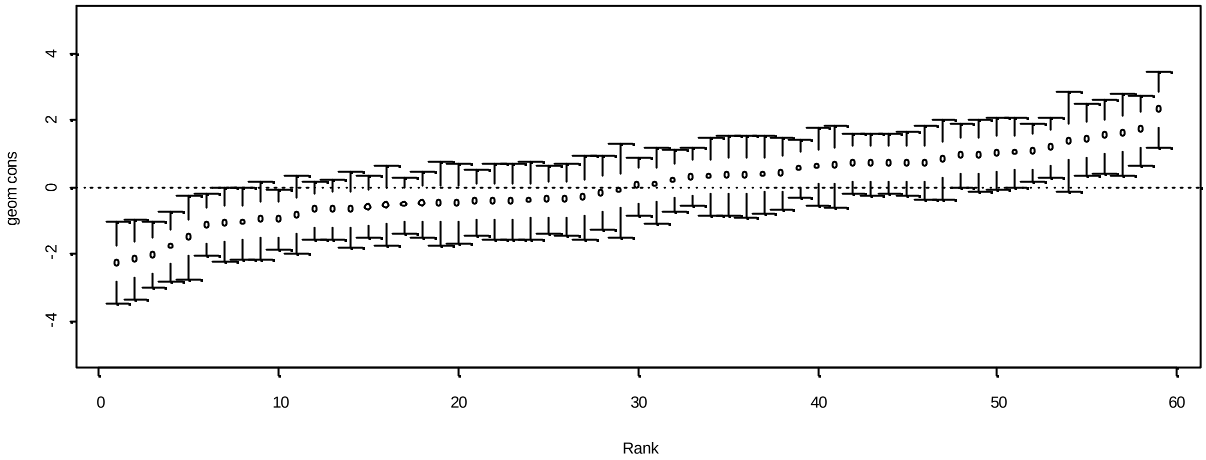
$$\begin{array}{c|cc} & Geo & Alg \\ \hline Geo & 2.173 & \\ Alg & r = .489 & 2.383 \end{array},$$

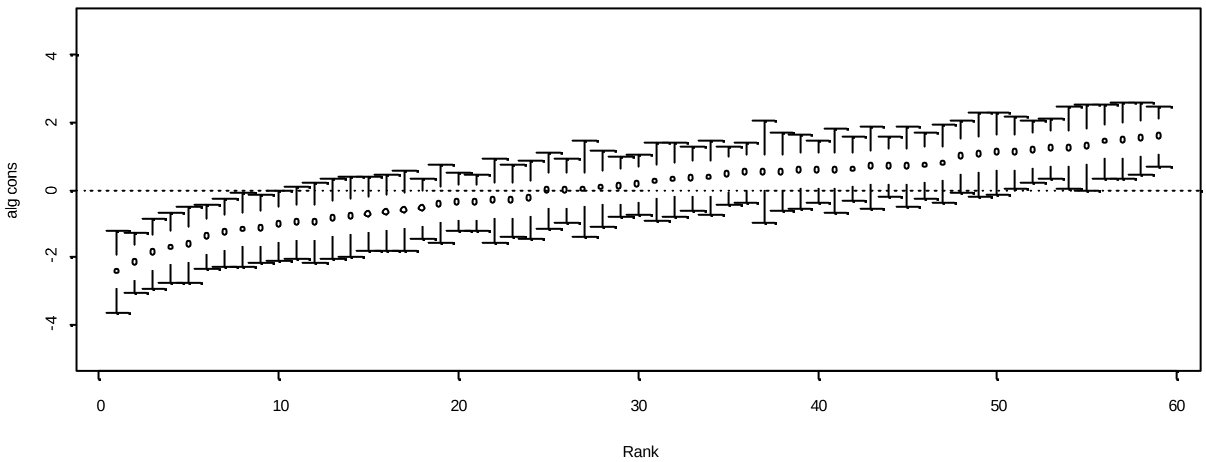
Student level (variances on the diagonal; correlations elsewhere)

		Girls		*Boys*	
		Geo	*Alg*	*Geo*	*Alg*
Girls	*Geo*	17.91			
	Alg	$r = .28$	19.02		
Boys	*Geo*			18.8	
	Alg			.35	21.99

which compares in an obvious way with that for Model 1. The residual variance/correlation matrix at student level is almost unchanged from Model 1. The full tabulation of the random part follows:

PARAMETER	ESTIMATE	S. ERROR	CORR.
School			
$\sigma_v^2(\text{geo\_cons})$	2.173	0.5167	1
$\sigma_v(\text{geo\_cons, alg\_cons})$	1.113	0.4203	0.509
$\sigma_v^2(\text{alg\_cons})$	2.383	0.5684	1
Student			
$\sigma_u^2(\text{geo\_girl})$	17.91	0.8378	1
$\sigma_u(\text{alg\_girl, geo\_girl})$	5.235	0.635	0.284
$\sigma_u^2(\text{alg\_girl})$	19.02	0.8915	1
$\sigma_u^2(\text{geo\_boy})$	18.8	0.8895	1
$\sigma_u(\text{alg\_boy, geo\_boy})$	7.025	0.7194	0.346
$\sigma_u^2(\text{alg\_boy})$	21.99	1.039	1


4
2
0
-2
-4
geom cons
0
10
20
30
40
50
60
Rank


4
2
0
-2
-4
alg cons
0
10
20
30
40
50
60
Rank

School Residual Ranks from Model 3

School	Geometry	Algebra
1	56	56
2	44	53
3	8	11
4	7	34
5	40	47
6	6	6
7	32	52
8	18	46
9	58	10
10	14	28
11	25	3
12	30	35
13	23	16
14	12	2
15	20	12
16	22	5
17	49	48
18	9	13
20	52	29
21	48	42
22	26	15
23	3	7
24	42	59
25	45	26
26	38	39
27	57	49
28	50	51
29	2	45
30	59	57
31	34	31
32	11	24
33	37	19
34	51	23
35	10	30
36	5	1
37	35	17
39	29	27
40	16	14
41	28	8
42	21	9
43	47	54
44	17	21
45	39	20
46	19	22
48	55	58
50	1	41
51	46	32
52	15	40
53	36	50
54	4	4
55	31	25
56	41	43
57	43	33
58	13	18
59	33	36
60	54	37
61	24	38
62	53	44
64	27	55

Schools ranked according to their residuals for geometry (Model 3)

School	Geometry	Algebra
30	59	57
9	58	10
27	57	49
1	56	56
48	55	58
60	54	37
62	53	44
20	52	29
34	51	23
28	50	51
17	49	48
21	48	42
43	47	54
51	46	32
25	45	26
2	44	53
57	43	33
24	42	59
56	41	43
5	40	47
45	39	20
26	38	39
33	37	19
53	36	50
37	35	17
31	34	31
59	33	36
7	32	52
55	31	25
12	30	35
39	29	27
41	28	8
64	27	55
22	26	15
11	25	3
61	24	38
13	23	16
16	22	5
42	21	9
15	20	12
46	19	22
8	18	46
44	17	21
40	16	14
52	15	40
10	14	28
58	13	18
14	12	2
32	11	24
35	10	30
18	9	13
3	8	11
4	7	34
6	6	6
36	5	1
54	4	4
23	3	7
29	2	45
50	1	41

Schools ranked according to their residuals for algebra (Model 3)

School	Geometry	Algebra
24	42	59
48	55	58
30	59	57
1	56	56
64	27	55
43	47	54
2	44	53
7	32	52
28	50	51
53	36	50
27	57	49
17	49	48
5	40	47
8	18	46
29	2	45
62	53	44
56	41	43
21	48	42
50	1	41
52	15	40
26	38	39
61	24	38
60	54	37
59	33	36
12	30	35
4	7	34
57	43	33
51	46	32
31	34	31
35	10	30
20	52	29
10	14	28
39	29	27
25	45	26
55	31	25
32	11	24
34	51	23
46	19	22
44	17	21
45	39	20
33	37	19
58	13	18
37	35	17
13	23	16
22	26	15
40	16	14
18	9	13
15	20	12
3	8	11
9	58	10
42	21	9
41	28	8
23	3	7
6	6	6
16	22	5
54	4	4
11	25	3
14	12	2
36	5	1

School Outliers according to geometry

NAME	COL1	COL2	COL3	COL4	COL5	COL6	COL7	COL8
Above the mean for geometry	30	9	27	1	48	62	20	.
Below the mean for geometry	35	3	6	36	54	23	29	50

School Outliers according to algebra

NAME	COL1	COL2	COL3	COL4	COL5	COL6	COL7	COL8	COL9	COL10
Above the mean for algebra	24	48	30	1	43	2	7	28	.	.
Below the mean for algebra	9	42	41	23	6	16	54	11	14	36

School Outliers according to difference between geometry and algebra

NAME	COL1	COL2	COL3	COL4	COL5
Large differences between rankings on geometry and algebra	9	11	14	50	29

4.6 Model 4

Model 4 (as Model 2 but with no random effect of gender at school level)

The model for the fixed part is:

$$\text{predicted geometry score} = 4.74 + 0.16\,BLMtest + 0.04\,BLMtest^2 + 1.02\,girl + 0.04\,s\%A\_C,$$

$$\text{predicted algebra score} = 8.64 - 0.31\,BLMtest + 0.03\,BLMtest^2 + 1.26\,girl + 0.22\,tage + 1.29\,tHE + 1.07\,stext,$$

similar to Model 2. Standard errors are as in the table below:

parameter	estimate	s. error(u)
alg_cons	8.637	1.651
alg_girl	1.264	0.2188
alg_BLMtest	-0.3107	0.2079
alg_BLMtest2	0.03277	0.006826
alg_tage	0.2249	0.1253
alg_tHE	1.291	0.5046
alg_stext	1.066	0.4236
geo_cons	4.737	1.595
geo_girl	1.002	0.2073
geo_BLMtest	0.1617	0.198
geo_BLMtest2	0.01421	0.006498
geo_s%A_C	0.04111	0.01148

The estimated residual variance/correlation matrix at school level is:

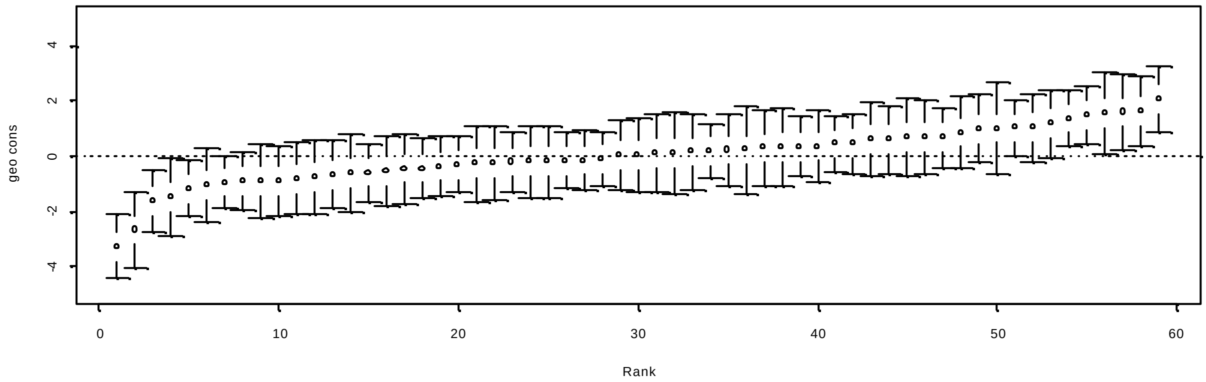
$$\begin{array}{c|cc} & G & A \\ \hline G & 1.561 & \\ A & r = .301 & 2.1 \end{array}.$$

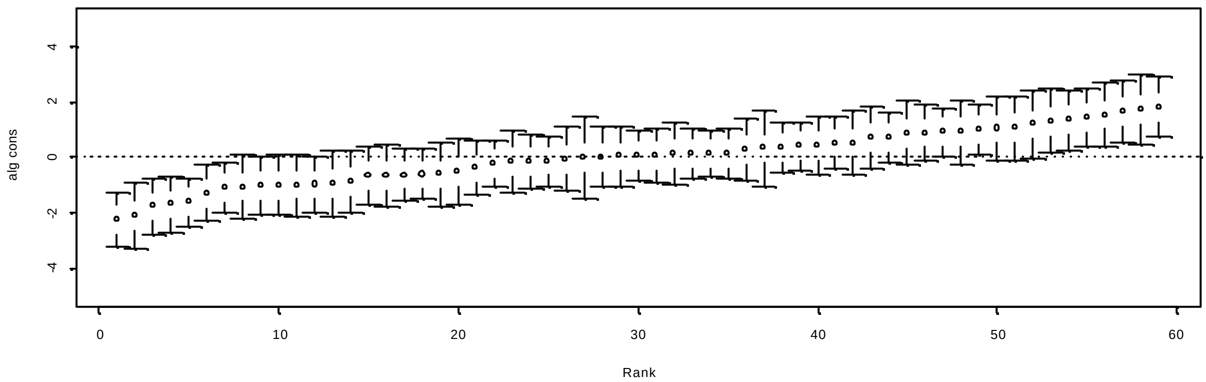
Student level (variances on the diagonal; correlations elsewhere)

		Girls		*Boys*	
		Geo	*Alg*	*Geo*	*Alg*
Girls	*Geo*	17.85			
	Alg	$r = .28$	18.9		
Boys	*Geo*			18.85	
	Alg			.35	21.93

Compared to Model 2, it is relatively straightforward to detect correlation at school level between performance in Geometry and in Algebra. Statistical test shows that there is no difference in variance between boys' Algebra and girls'. The full tabulation of the random part is:

PARAMETER	ESTIMATE	S. ERROR	CORR.
School			
$\sigma_v^2(\text{geo\_cons})$	1.561	0.4027	1
$\sigma_v(\text{geo\_cons, alg\_cons})$	0.5453	0.336	0.301
$\sigma_v^2(\text{alg\_cons})$	2.1	0.5153	1
Student			
$\sigma_u^2(\text{geo\_girl})$	17.85	0.8344	1
$\sigma_u(\text{alg\_girl, geo\_girl})$	5.199	0.6319	0.283
$\sigma_u^2(\text{alg\_girl})$	18.9	0.8853	1
$\sigma_u^2(\text{geo\_boy})$	18.85	0.8913	1
$\sigma_u(\text{alg\_boy, geo\_boy})$	7.071	0.7194	0.348
$\sigma_u^2(\text{alg\_boy})$	21.93	1.036	1


geo cons
4
2
0
-2
-4
0
10
20
30
40
50
60
Rank


alg cons
4
2
0
-2
-4
0
10
20
30
40
50
60
Rank

School Residual Ranks from Model 4

School	Geometry	Algebra
1	45	52
2	44	44
3	4	11
4	9	30
5	50	50
6	6	7
7	25	48
8	15	31
9	59	5
10	8	10
11	37	4
12	38	45
13	10	15
14	22	3
15	29	17
16	33	8
17	48	59
18	7	12
20	47	23
21	41	40
22	28	13
23	3	6
24	40	57
25	35	39
26	23	35
27	57	42
28	46	47
29	5	53
30	55	37
31	30	41
32	20	29
33	31	24
34	54	18
35	13	32
36	16	1
37	49	21
39	34	38
40	32	20
41	11	16
42	24	9
43	39	51
44	12	28
45	43	25
46	21	19
48	51	55
50	1	54
51	52	26
52	19	36
53	26	58
54	2	2
55	27	22
56	36	43
57	53	34
58	18	14
59	42	33
60	56	27
61	17	46
62	58	49
64	14	56

Schools ranked according to their residuals for geometry (Model 4)

School	Geometry	Algebra
9	59	5
62	58	49
27	57	42
60	56	27
30	55	37
34	54	18
57	53	34
51	52	26
48	51	55
5	50	50
37	49	21
17	48	59
20	47	23
28	46	47
1	45	52
2	44	44
45	43	25
59	42	33
21	41	40
24	40	57
43	39	51
12	38	45
11	37	4
56	36	43
25	35	39
39	34	38
16	33	8
40	32	20
33	31	24
31	30	41
15	29	17
22	28	13
55	27	22
53	26	58
7	25	48
42	24	9
26	23	35
14	22	3
46	21	19
32	20	29
52	19	36
58	18	14
61	17	46
36	16	1
8	15	31
64	14	56
35	13	32
44	12	28
41	11	16
13	10	15
4	9	30
10	8	10
18	7	12
6	6	7
29	5	53
3	4	11
23	3	6
54	2	2
50	1	54

Schools ranked according to their residuals for algebra (Model 4)

School	Geometry	Algebra
17	48	59
53	26	58
24	40	57
64	14	56
48	51	55
50	1	54
29	5	53
1	45	52
43	39	51
5	50	50
62	58	49
7	25	48
28	46	47
61	17	46
12	38	45
2	44	44
56	36	43
27	57	42
31	30	41
21	41	40
25	35	39
39	34	38
30	55	37
52	19	36
26	23	35
57	53	34
59	42	33
35	13	32
8	15	31
4	9	30
32	20	29
44	12	28
60	56	27
51	52	26
45	43	25
33	31	24
20	47	23
55	27	22
37	49	21
40	32	20
46	21	19
34	54	18
15	29	17
41	11	16
13	10	15
58	18	14
22	28	13
18	7	12
3	4	11
10	8	10
42	24	9
16	33	8
6	6	7
23	3	6
9	59	5
11	37	4
14	22	3
54	2	2
36	16	1

School Outliers according to geometry

NAME	COL1	COL2	COL3	COL4	COL5	COL6
Above the mean for geometry	9	62	27	57	.	.
Below the mean for geometry	6	29	3	23	54	50

School Outliers according to algebra

NAME	COL1	COL2	COL3	COL4	COL5	COL6	COL7	COL8	COL9
Above the mean for algebra	17	53	24	48	29	1	62	7	.
Below the mean for algebra	42	16	6	23	9	11	14	54	36

School Outliers according to difference between geometry and algebra

NAME	COL1	COL2	COL3	COL4	COL5	COL6
Large differences between rankings on geometry and algebra	9	11	34	14	29	50

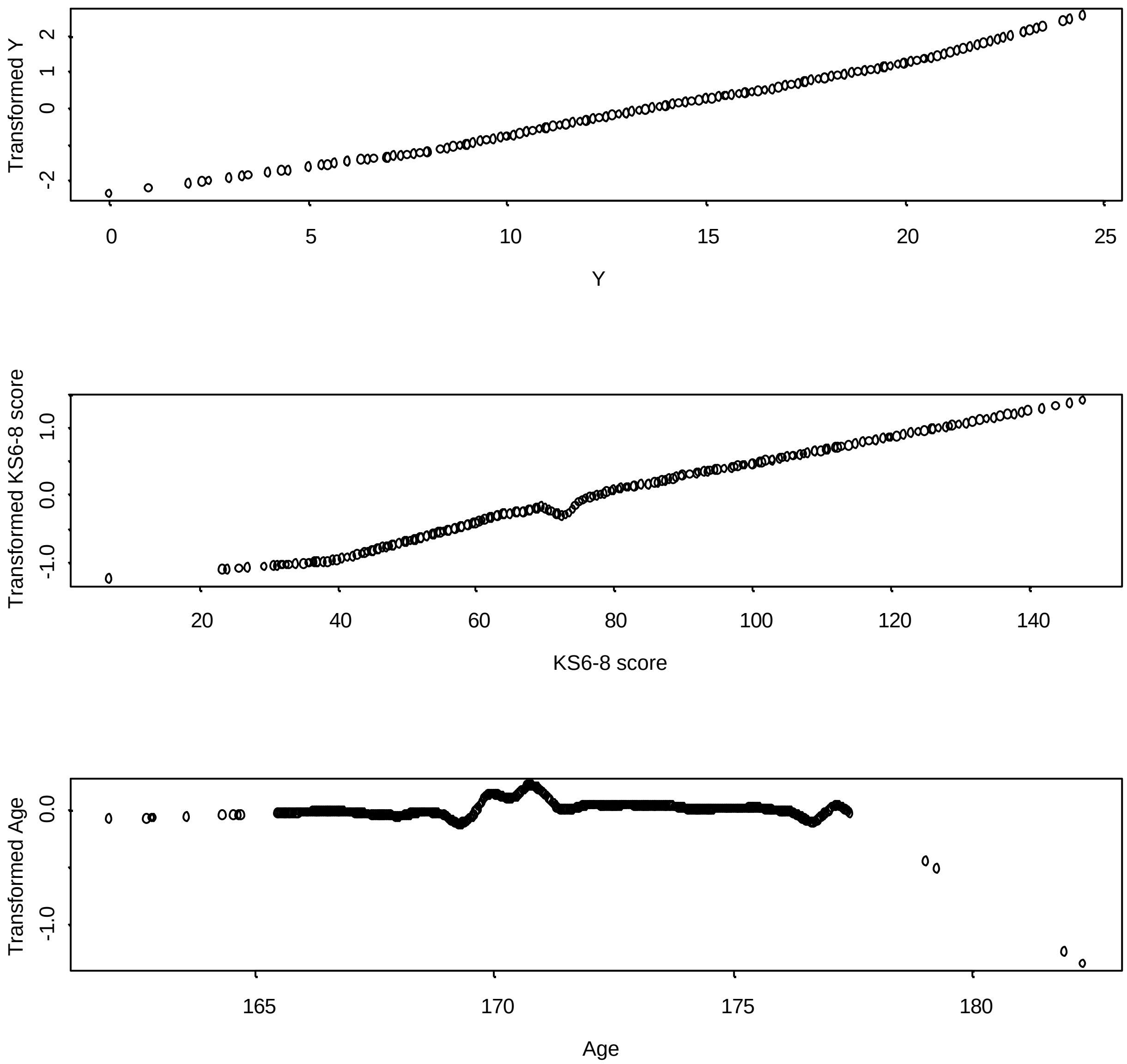
Largest changes in rankings from model 3 to model 4

School ID	Change in geometry	School ID	Change in algera
41	-17	30	-20
26	-15	10	-18
13	-13	8	-15
11	12	39	11
37	14	25	13
40	16	50	13

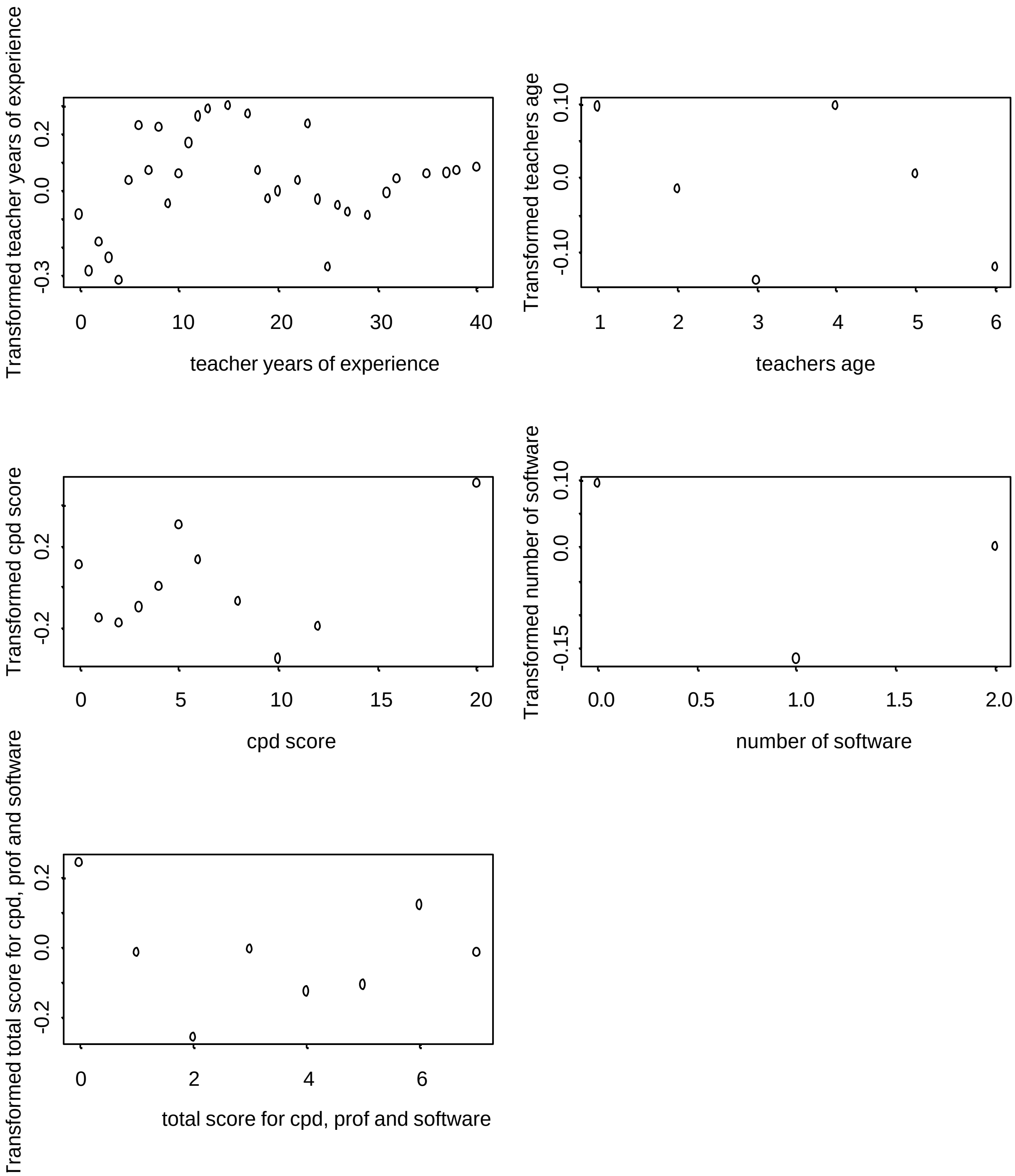
5 Modeling of the Year 9 Proof Score using the Key Stage 3 Test Score as Baseline

5.1 Variable Codings as suggested by ACE algorithm

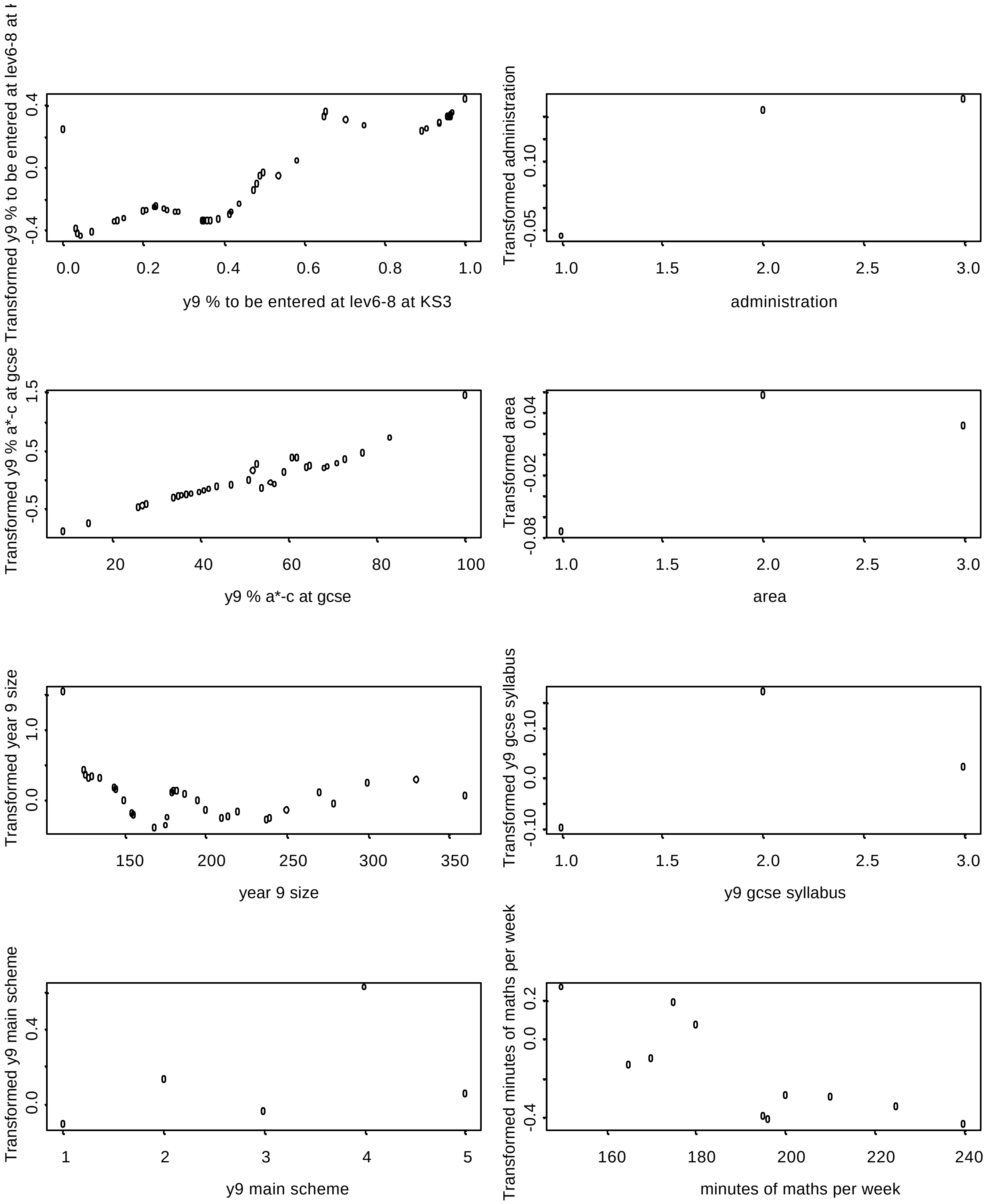
After application of ACE algorithm, the following ACE plots for various variables are produced and displayed in the following fix pages.



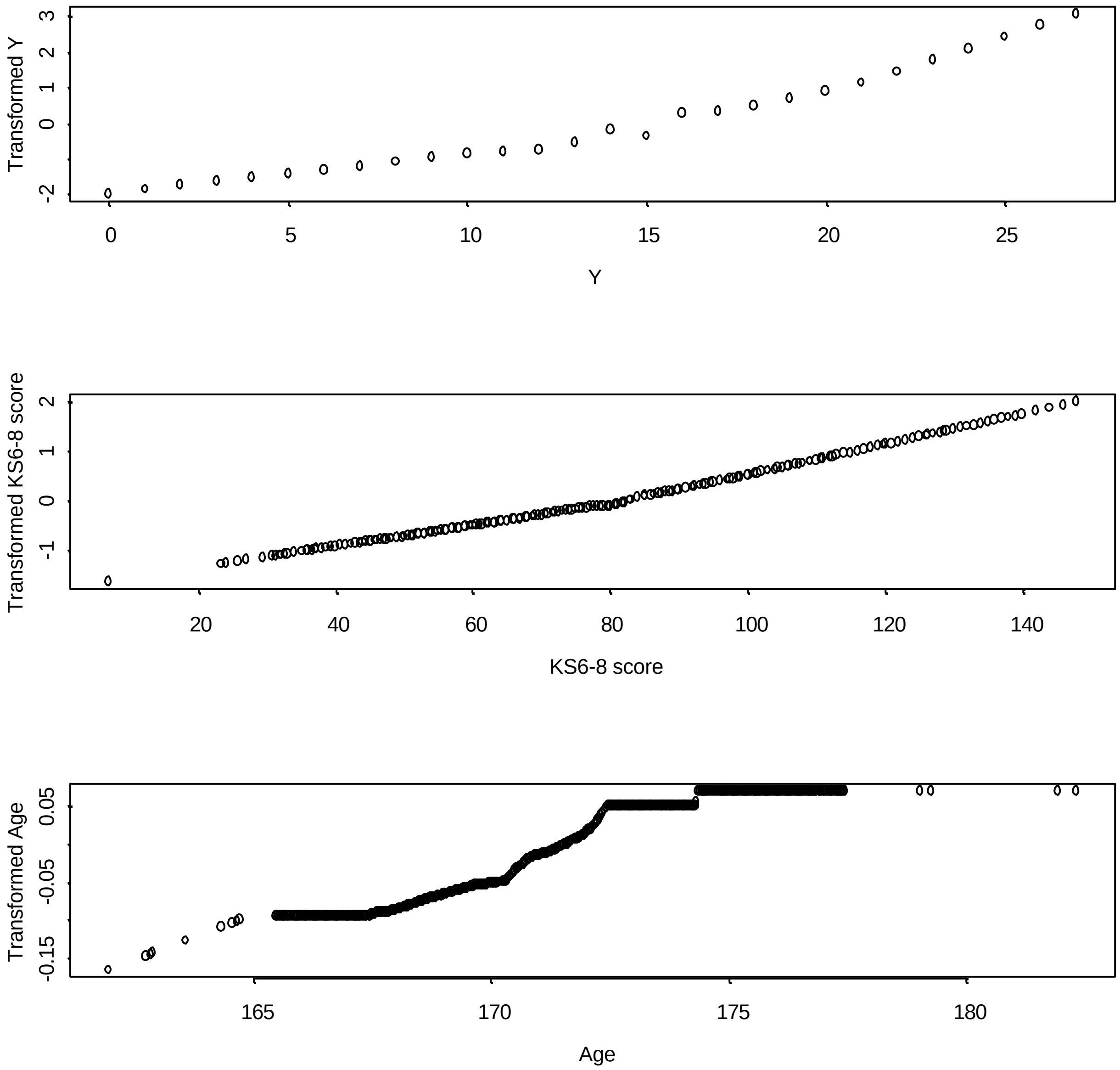
ACE plots for geometry analysis



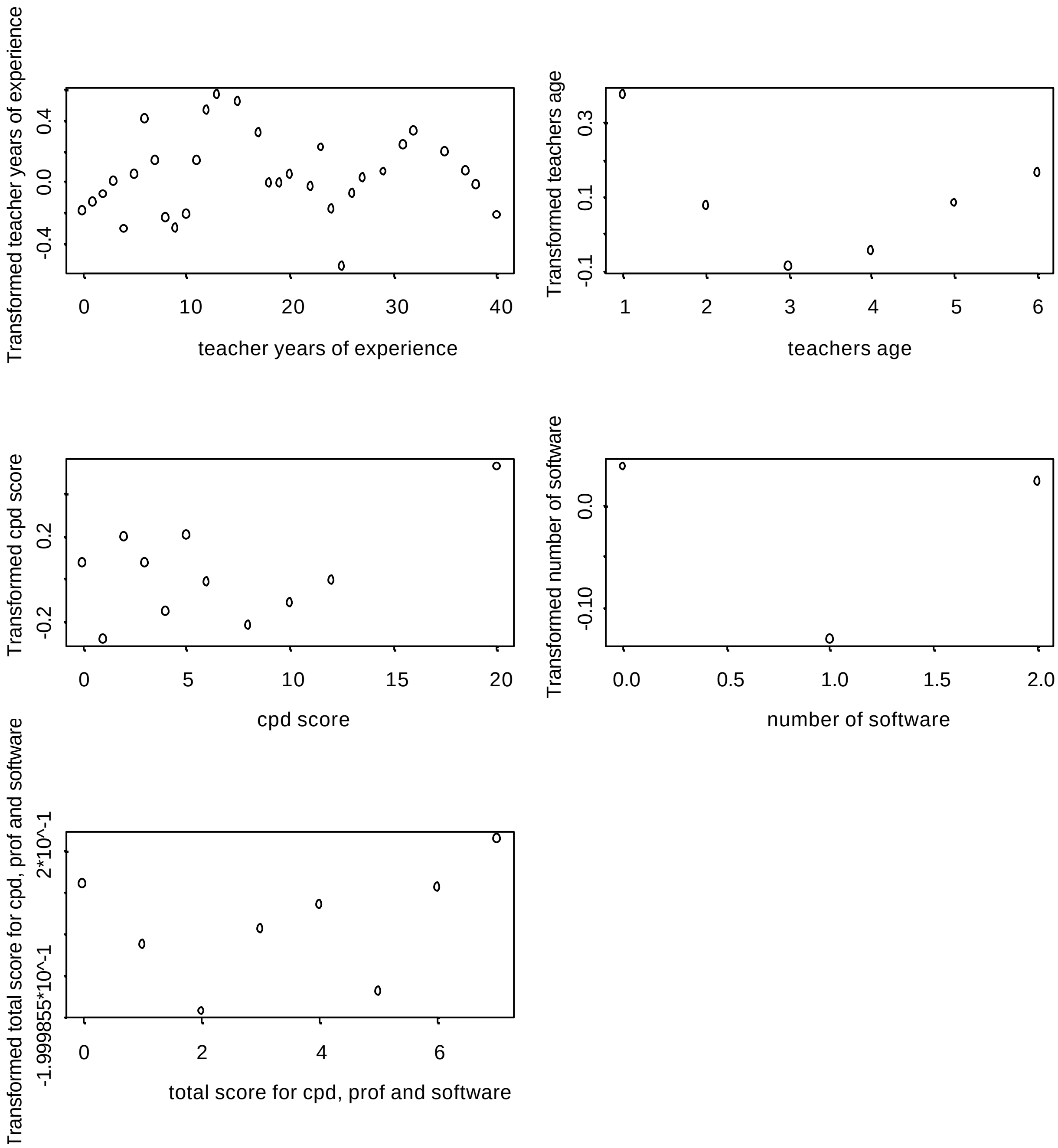
ACE plots for geometry analysis



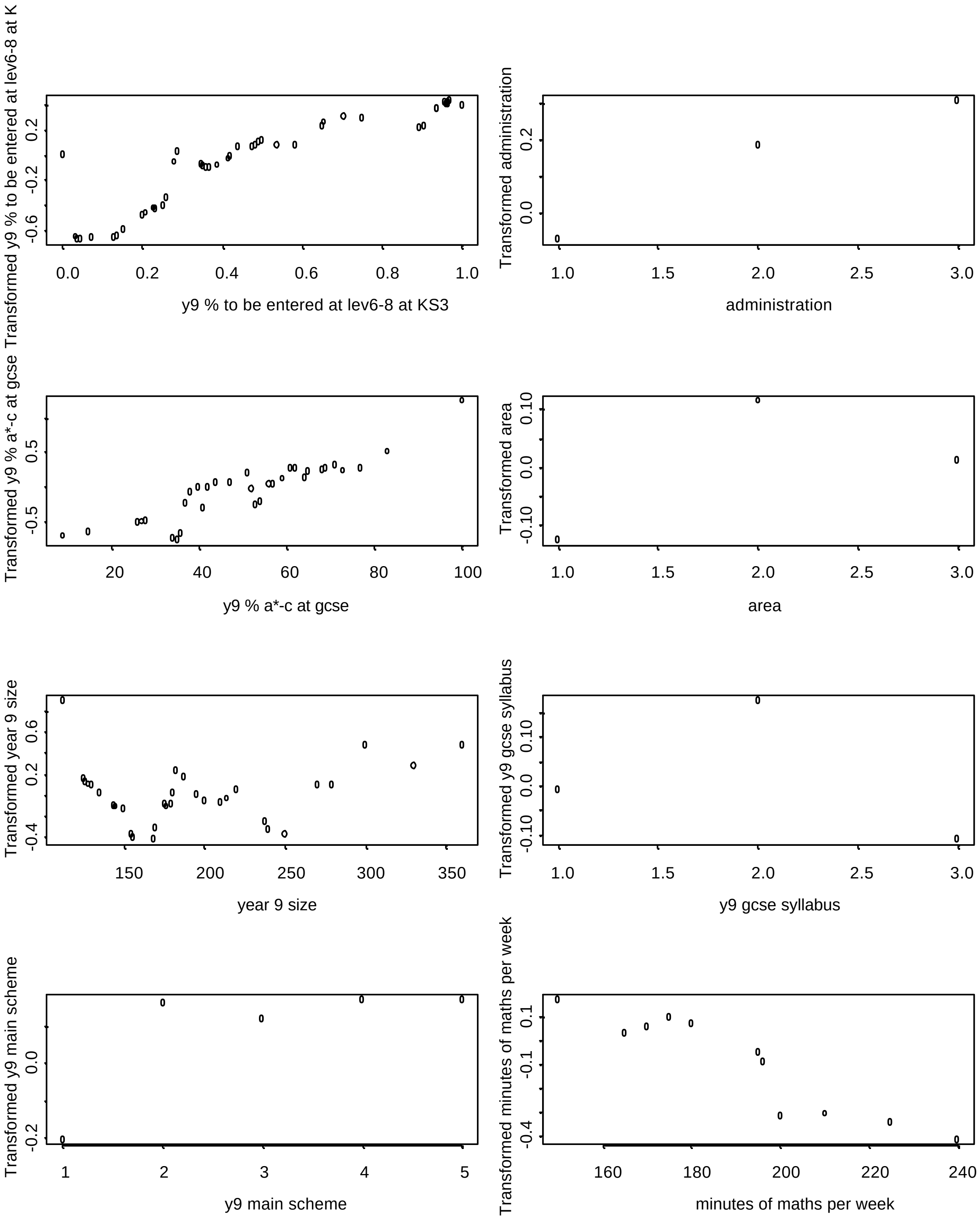
ACE plots for geometry analysis



ACE plots for algebra analysis



ACE plots for algebra analysis



ACE plots for algebra analysis

Based on the ACE plots above and BIC statistic calculation, we decided the following transformation or codings for the variable in the analysis:

Variables	Name	Variable Transformation or Codings
Response Variable		
Year 9 proof score	YR9prf	linear
Predictor Variables		
Key Stage 3 test score	KS3test	linear score
school-level:		
administration	sadmin	0=county, 1=voluntary (VA/VC) or other
age range 11-16 or 11-18	sage	1=11-16, 0=11-18
% A*-C at GCSE	s%A_C	linear
gender	smix	1=girls only, 0=mixed
area (one of three)	sarea	1=urban or suburban, 0=rural
year-9 size	ssize	linear(school size)
% to be entered at lev6-8 at KS3	spent1	min(Year 9 % enter at KS3, 15%)
	spent2	max(0,min(Year 9 % at KS3-15%,25%-15%))"
	spent3	max(0,Year 9 % enter at KS3-25%)
GCSE syllabus	ssyll	1=OCR,0=Others
maths textbook or scheme in use	stext	1=SMP/Vickers/ST(P)/other,0=Key Math
minutes of maths per week	smtime	linear (minutes)
existence of maths club	smclub	1=Yes,0=No
class-level:		
teacher gender	tsex	1=Female, 0=Male
teacher years of experience	tyearex1	min(year,12)
	tyearex2	max(0,min(year-12,22-12)
	tyearex3	max(0,year-22)
teacher age	tage	linear(year)
teacher degree	tdegree	1=having a maths degree, 0=otherwise
teacher PGCE or Cert	tpgrt	1=having a PGCE or Cert,0=otherwise
teacher HE	tHE	1=having MSc or PhD degreee, 0=otherwise
teacher CPD score	tcpd	linear(score)
teacher membership of a professional assoc.	tmember	1=Yes, 0=No
teacher knowledge/use of software	tsoft	1=Yes,0=No
teacher total score for CPD, PROF and software	ttscore	linear(score)
student-level:		
gender	girl	1=Girl,0=Boy
age in months	age	linear(month)

5.2 Model 0

The fixed-part estimates from this model may be expressed by means of the two equations:

$$\text{predicted geometry score} = 13.48 + 0.35 girl,$$

$$\text{predicted algebra score} = 13.34 + 0.37 girl,$$

thus, the predicted score for the base group (boys) in Geometry is 13.54(s.e. 0.32) and in Algebra it is 14.35 (s.e. 0.36). The gender coefficients are statistically non-significant, as is shown in the full tabulation below:

parameter	estimate	s. error(u)
alg_cons	14.34	0.3435
alg_girl	0.3709	0.2368
geo_cons	13.48	0.2962
geo_girl	0.3542	0.223

Thus, there is *no* statistically significant gender effect on either outcome score when the model makes no adjustment for baseline score.

The residual variance/covariance matrices for the outcome scores at the three levels, school, class, and student, have the following estimates (covariances have been converted to correlations for convenience):

School level

	Geo	*Alg*
Geo	1.699	
Alg	$r = .875$	3.606

Class level

	Geo	*Alg*
Geo	2.802	
Alg	$r = .67$	2.175

Student level

	Geo	*Alg*
Geo	20.93	
Alg	$r = .40$	23.63

The full tabulation is:

PARAMETER	ESTIMATE	S. ERROR	CORR.
School			
σ^2_w(geo_cons)	1.699	0.9931	1
σ_w(geo_cons, alg_cons)	2.166	0.9525	0.875
σ^2_w(alg_cons)	3.606	1.25	1
Class			
σ^2_v(geo_cons)	2.802	0.9558	1
σ_v(geo_cons, alg_cons)	1.65	0.7516	0.668
σ^2_v(alg_cons)	2.175	0.8684	1
Student			
σ^2_u(geo_cons)	20.93	0.7008	1
σ_u(geo_cons, alg_cons)	8.973	0.568	0.404
σ^2_u(alg_cons)	23.63	0.7914	1

Two important observations can be found from the above results. First, there is statistically significant, though small, residual variation at class level, and high residual correlation ($r \approx .87$) at both school and class level between the two outcomes. Thus, in this very simple model, schools that perform above the average in algebra are predicted to do so in geometry also, and vice versa.

Second, class effects on the two subjects, within schools, are predicted to be similar. A student, however, who performs above the expectation for her class in algebra has only a slight tendency to perform above expectation in geometry also.

5.3 Model 1

In this analysis, we have included the Key Stage 3 (KS3test) test score in this model as a predictor. As suggested by the ACE algorithm, the relationship between year 9 proof score and KS3test score can be best described by a linear relationship as follows for geometry scores:

$$\text{predicted geometry score} = 4.30 + 0.11 KS3test + 0.82 girl \text{ ,}$$

The corresponding model for algebra scores is

$$\text{predicted algebra score} = 3.21 + 0.14 KS3test + 0.98 girl,$$

The full tabulation of the fixed part, showing the standard errors, is:

parameter	estimate	s. error(u)
alg_cons	3.21	0.4251
alg_girl	0.9831	0.2792
alg_KS3test	0.1357	0.0046
geo_cons	4.305	0.4482
geo_girl	0.822	0.227
geo_KS3test	0.1123	0.004639

The above results show that the gender has a significant effect on both geometry and algebra score.

Once baseline score is included, no statistically significant variation remains at class level within school. Thus, we have two levels of variation, students at level 2 within schools at level 3, with level 1 used as before to distinguish between the responses in Algebra and Geometry.

The relationship described by each equation is illustrated in the first of the two graphs on the following pages. The second graph shows the relationship of the outcome scores with the raw baseline score.

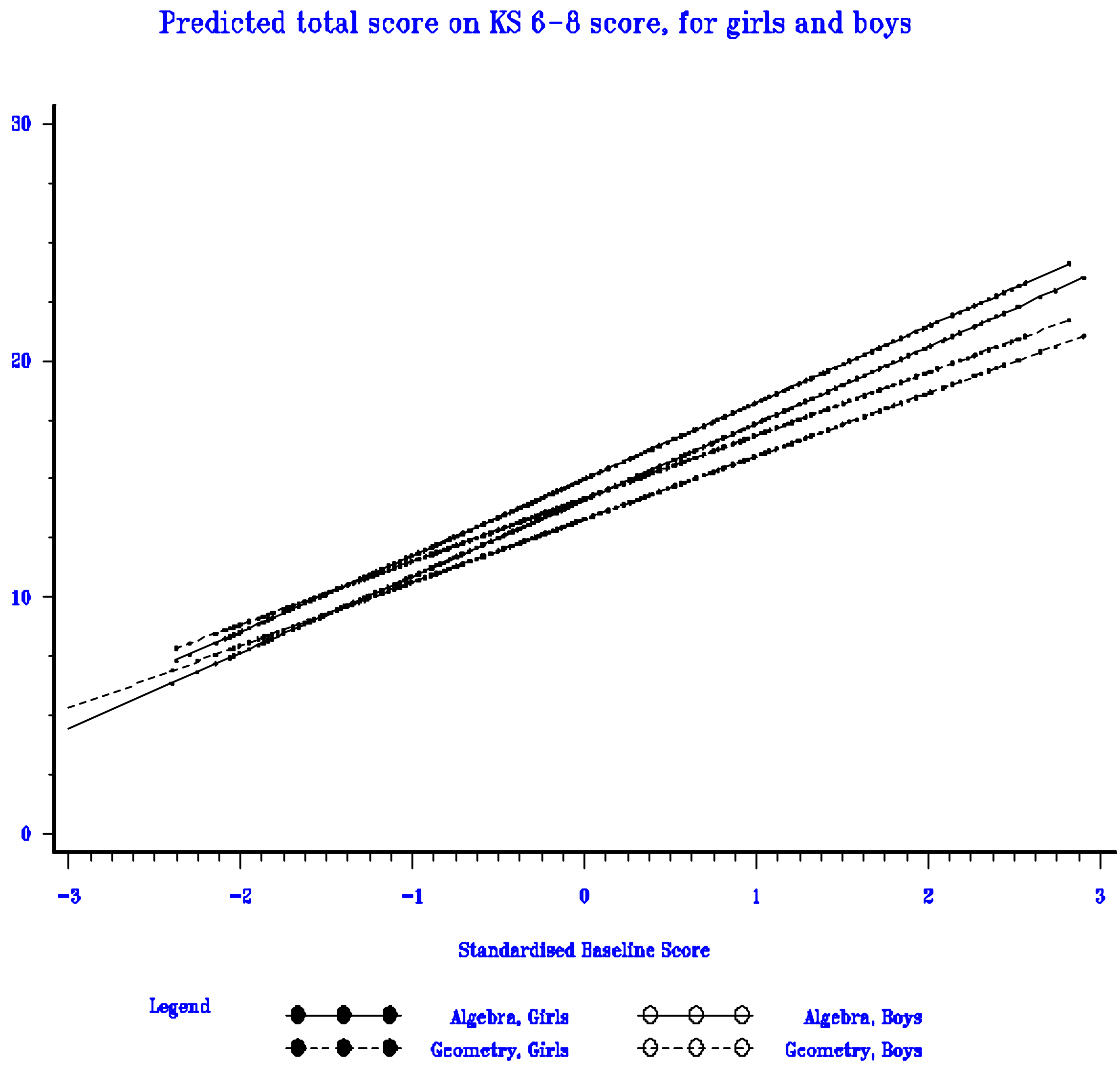
The random part of Model 1 can be expressed as follows:

School level (variances on the diagonal; correlations elsewhere)

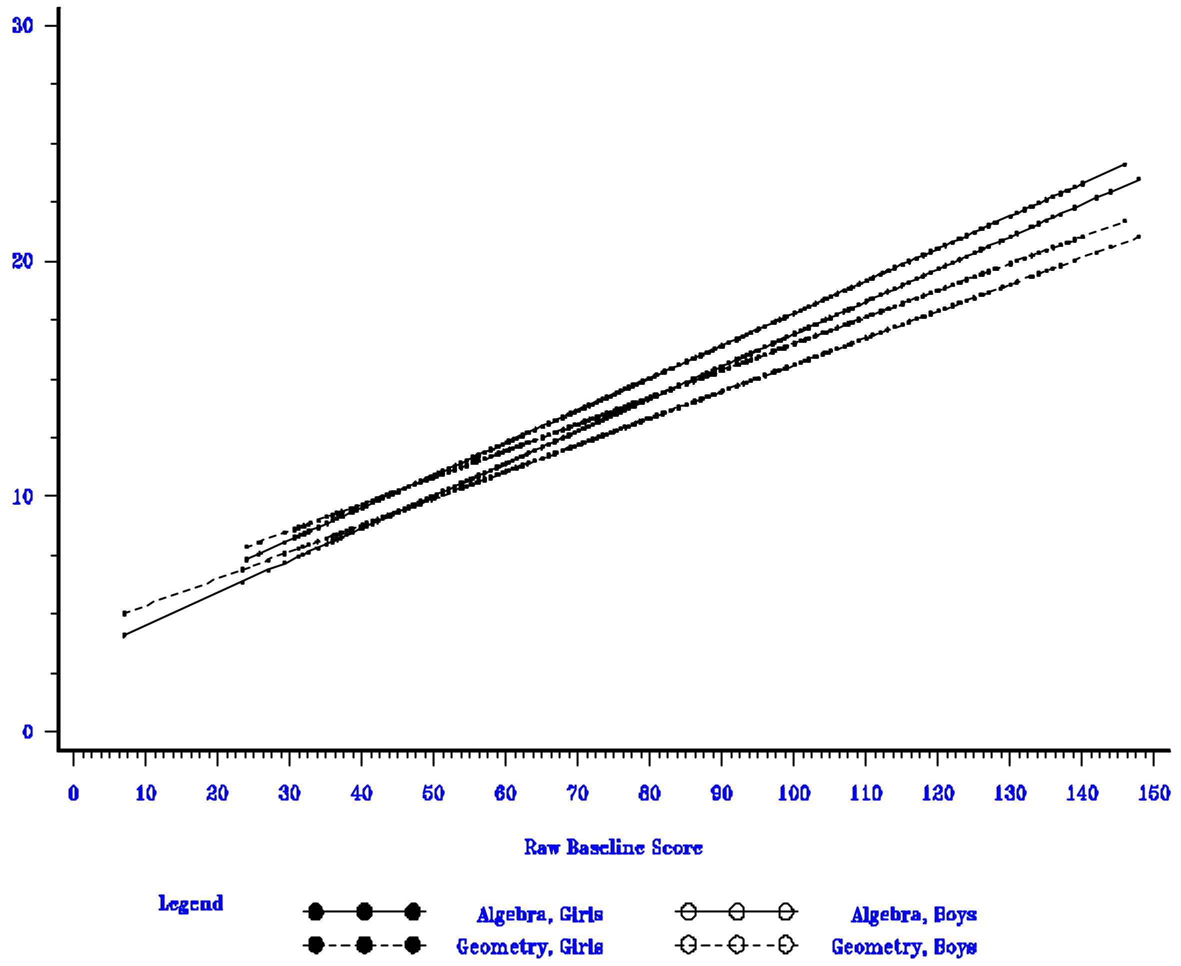
		Girls		*Boys*	
		Geo	*Alg*	*Geo*	*Alg*
Girls	*Geo*	1.379			
	Alg		1.059		
Boys	*Geo*	$r = .81$		1.844	
	Alg				0.963

Student level (variances on the diagonal; correlations elsewhere)

		Girls		*Boys*	
		Geo	*Alg*	*Geo*	*Alg*
Girls	*Geo*	17.8			
	Alg	$r = .29$	19.12		
Boys	*Geo*			18.55	
	Alg			.352	22.07


Predicted total score on KS 6-8 score, for girls and boys
30
20
10
0
-3
-2
-1
0
1
2
3
Standardised Baseline Score
Legend
Algebra, Girls
Geometry, Girls
Algebra, Boys
Geometry, Boys

Predicted total score on KS 6-8 score, for girls and boys

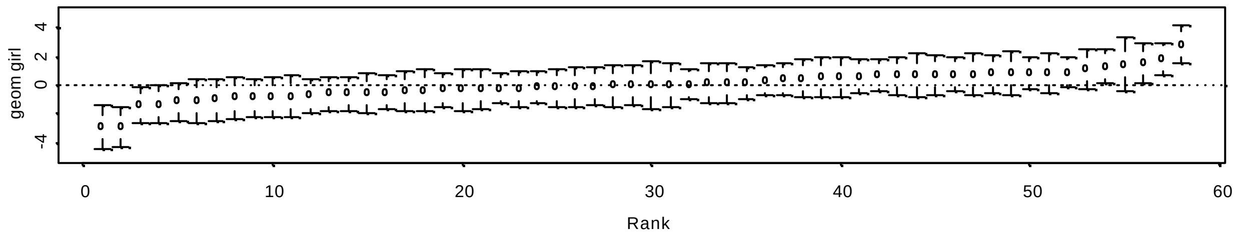


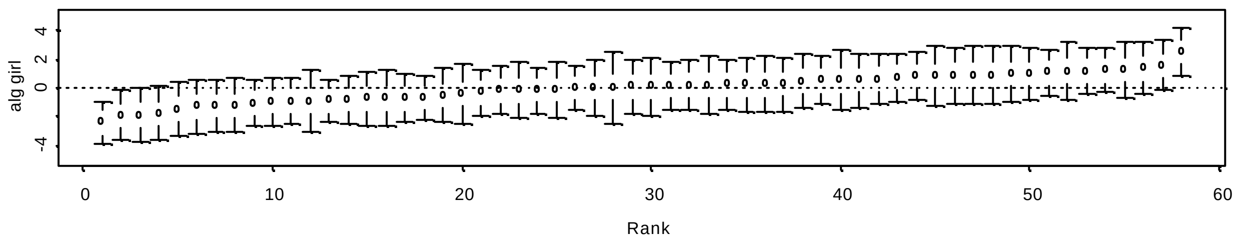
The full tabulation is:

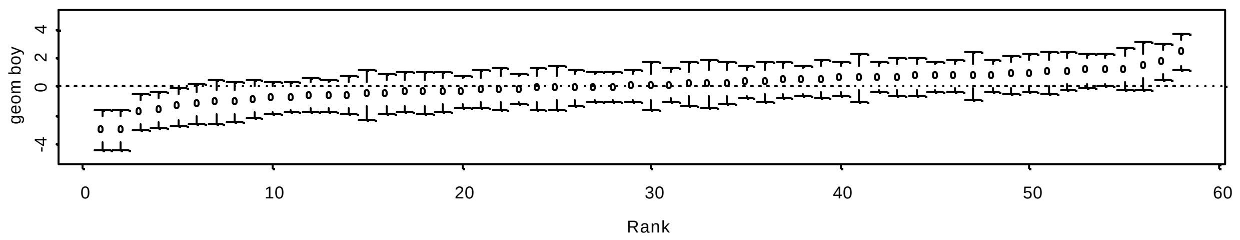
PARAMETER	ESTIMATE	S. ERROR	CORR.
School			
σ_v^2(geo_girl)	1.379	0.4495	1
σ_v^2(alg_girl)	1.059	0.3943	1
σ_v(geo_boy, geo_girl)	1.29	0.397	0.82
σ_v^2(geo_boy)	1.844	0.554	1
σ_v^2(alg_boy)	0.9634	0.4017	1
Student			
σ_u^2(geo_girl)	16.46	0.7781	1
σ_u(alg_girl, geo_girl)	3.274	0.5622	0.197
σ_u^2(alg_girl)	16.76	0.7933	1
σ_u^2(geo_boy)	17.06	0.8168	1
σ_u^2(alg_boy)	18.81	0.8994	1
s_u(alg_boy, geo_boy)	4.671	0.6223	0.261

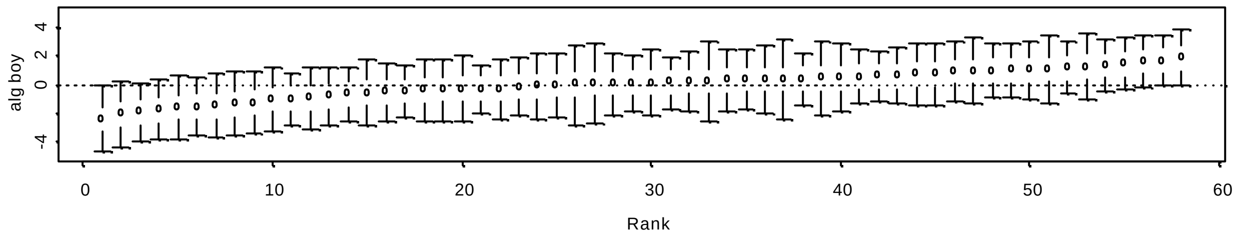
The very high value of correlation between girls' and boys' geometry scores (within subject) at school level mean that, for example, a school that has a high residual for boys is very likely to have a high residual also for girls in the same subject.

The following chart plots school-level residuals against their ranks, with error bars corresponding to 1.96 SD. Thus, an error bar wholly above the dotted line corresponds to a school that is performing above the mean predicted by the model, with 95% confidence.









Tables of the school residual ranks now follow (A high-numbered rank indicates good performance.)

School Residual Ranks from Model 1

School	geo_girl	alg_girl	alg_boy	geo_boy
1	43	35	.	.
2	42	56	52	33
3	5	7	5	4
4	4	25	40	5
5	32	43	.	.
6	30	24	9	21
7	21	47	43	18
8	18	37	51	12
9	58	6	13	54
10	11	51	22	20
11	38	2	7	29
12	36	30	53	26
13	26	49	6	37
14	45	1	14	31
15	19	29	10	35
16	20	5	1	27
17	46	44	41	44
18	8	13	12	9
20	57	15	54	51
21	47	33	35	43
22	41	34	28	39
23	3	10	16	3
24	15	55	42	24
25	40	11	26	30
26	31	50	23	36
27	52	48	18	50
28	51	54	36	47
29	1	39	50	2
30	53	19	.	.
31	33	23	.	.
32	10	16	44	7
33	44	9	21	34
34	56	32	19	53
35	16	57	39	8
36	13	3	4	23
37	49	27	15	48
39	22	12	31	16
40	34	20	25	22
41	29	4	2	14
42	12	8	3	15
43	9	42	8	19
44	23	18	20	11
45	48	17	17	41
46	27	22	32	25
50	2	36	33	1
51	50	38	27	45
52	24	58	47	28
53	17	45	49	13
54	14	26	11	10
55	39	21	46	42
56	25	28	24	32
57	54	40	48	49
58	37	53	37	38
59	28	46	38	40
60	55	31	29	52
61	6	41	30	17
62	35	14	34	46
64	7	52	45	6

Schools ranked according to their residuals for girls geometry (Model 1)

School	geo_girl	alg_girl	alg_boy	geo_boy
9	58	6	13	54
20	57	15	54	51
34	56	32	19	53
60	55	31	29	52
57	54	40	48	49
30	53	19	.	.
27	52	48	18	50
28	51	54	36	47
51	50	38	27	45
37	49	27	15	48
45	48	17	17	41
21	47	33	35	43
17	46	44	41	44
14	45	1	14	31
33	44	9	21	34
1	43	35	.	.
2	42	56	52	33
22	41	34	28	39
25	40	11	26	30
55	39	21	46	42
11	38	2	7	29
58	37	53	37	38
12	36	30	53	26
62	35	14	34	46
40	34	20	25	22
31	33	23	.	.
5	32	43	.	.
26	31	50	23	36
6	30	24	9	21
41	29	4	2	14
59	28	46	38	40
46	27	22	32	25
13	26	49	6	37
56	25	28	24	32
52	24	58	47	28
44	23	18	20	11
39	22	12	31	16
7	21	47	43	18
16	20	5	1	27
15	19	29	10	35
8	18	37	51	12
53	17	45	49	13
35	16	57	39	8
24	15	55	42	24
54	14	26	11	10
36	13	3	4	23
42	12	8	3	15
10	11	51	22	20
32	10	16	44	7
43	9	42	8	19
18	8	13	12	9
64	7	52	45	6
61	6	41	30	17
3	5	7	5	4
4	4	25	40	5
23	3	10	16	3
50	2	36	33	1
29	1	39	50	2

Schools ranked according to their residuals for girls algebra (Model 1)

School	geo_girl	alg_girl	alg_boy	geo_boy
52	24	58	47	28
35	16	57	39	8
2	42	56	52	33
24	15	55	42	24
28	51	54	36	47
58	37	53	37	38
64	7	52	45	6
10	11	51	22	20
26	31	50	23	36
13	26	49	6	37
27	52	48	18	50
7	21	47	43	18
59	28	46	38	40
53	17	45	49	13
17	46	44	41	44
5	32	43	.	.
43	9	42	8	19
61	6	41	30	17
57	54	40	48	49
29	1	39	50	2
51	50	38	27	45
8	18	37	51	12
50	2	36	33	1
1	43	35	.	.
22	41	34	28	39
21	47	33	35	43
34	56	32	19	53
60	55	31	29	52
12	36	30	53	26
15	19	29	10	35
56	25	28	24	32
37	49	27	15	48
54	14	26	11	10
4	4	25	40	5
6	30	24	9	21
31	33	23	.	.
46	27	22	32	25
55	39	21	46	42
40	34	20	25	22
30	53	19	.	.
44	23	18	20	11
45	48	17	17	41
32	10	16	44	7
20	57	15	54	51
62	35	14	34	46
18	8	13	12	9
39	22	12	31	16
25	40	11	26	30
23	3	10	16	3
33	44	9	21	34
42	12	8	3	15
3	5	7	5	4
9	58	6	13	54
16	20	5	1	27
41	29	4	2	14
36	13	3	4	23
11	38	2	7	29
14	45	1	14	31

School Outliers according to geometry

NAME	COL1	COL2	COL3	COL4
Above the mean for girls geometry	9	20	34	57
Above the mean for boys geometry	9	34	.	.
Below the mean for girls geometry	4	23	50	29
Below the mean for boys geometry	3	23	29	50

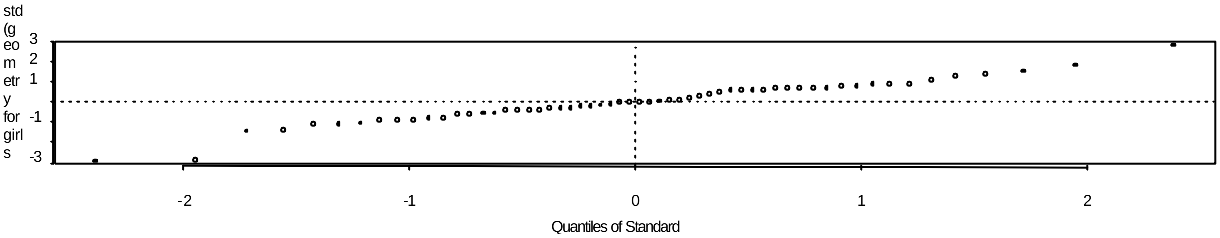
School Outliers according to algebra

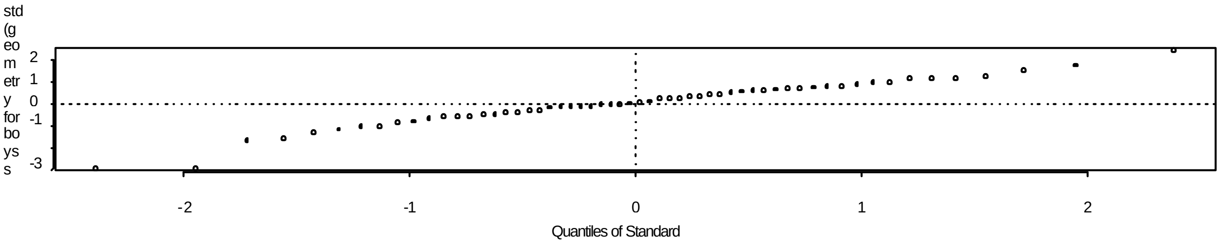
NAME	COL1	COL2
Above the mean for girls algebra	52	.
Above the mean for boys algebra	.	.
Below the mean for girls algebra	11	14
Below the mean for boys algebra	.	.

School Outliers according to difference between geometry and algebra

NAME	COL1	COL2	COL3	COL4	COL5	COL6
Large differences between rankings on girls geometry and algebra	9	14	20	52	50	29
Large differences between rankings on boys geometry and algebra	29	34	50	9	.	.

We also plotted standardised diagnostic school-level residuals against their normal scores (see below). These are not ideal, and reflect problems in the scoring of the outcome.


std (g eo m etr y for girl s
3
2
1
-1
-3
-2
-1
0
1
2
Quantiles of Standard


std (g eo m etr y for bo ys s
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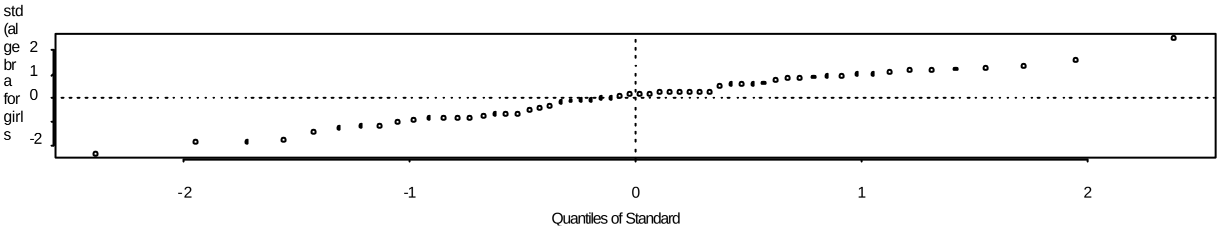
Quantiles of Standard


std (al ge br a for girl s
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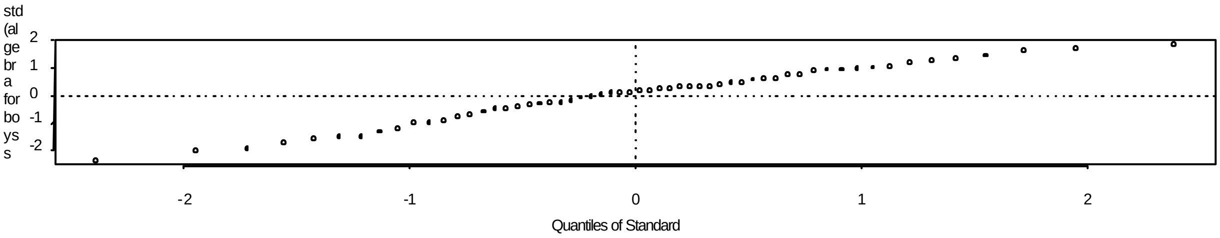
Quantiles of Standard


std (al ge br a for bo ys s
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1
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-1
-2
-2
-1
0
1
2
Quantiles of Standard

5.4 Model 2

Model 2 is the most parsimonious model selected by forward and backward procedure.

The fixed part of Model 2 is:

$$\text{predicted geometry score} = 3.44 + 0.11 KS3test + 0.81 girl + 0.02 \times s\%A\_C,$$

$$\text{predicted algebra score} = 1.66 + 0.13 KS3test + 1.04 girl + 0.27 tage + 1.01 tHE + 1.00 \times stext,$$

The fixed-part parameters are tabulated, with their standard errors, on the next page.

```
parameter            estimate     s. error(u)
alg_cons                1.657       0.6015
alg_girl                1.041       0.2603
alg_KS3test            0.1341     0.004529
alg_tage               0.2682       0.1048
alg_tHE                  1.01        0.399
alg_stext              0.9983       0.2622

geo_cons                3.445       0.6419
geo_girl               0.8097       0.2266
geo_KS3test            0.1094     0.004777
geo_s%A_C             0.02111      0.01113
```

The results show that s%A_C (the school % GCSE pass rate at A*-C) statistically significantly associated with the geometry score: 10% increase in this variable will result in .2 proof score.

Three additional variables are found to be significant predictors of proof algebra score. A teacher with a MSc or PhD degree education will on average increase the proof score by 1.01 and students using of textbooks other than “Key Math“ will have 1.00 higher proof score than those using other textbooks. In addition, the older the teacher the higher the proof score.

Turning to the random part of Model 2, we find that including the additional predictor in the fixed part reduces the school-level variation. Schools’ residual performance in algebra is significantly more variable for girls than for boys. There is little change from Model 1 in the random part at student level.

School level (variances on the diagonal; correlations elsewhere)

		Girls		*Boys*	
		Geo	*Alg*	*Geo*	*Alg*
Girls	*Geo*	1.236			
	Alg	$r = .80$	0.753		
Boys	*Geo*			1.901	
	Alg				0.826

Student level (variances on the diagonal; correlations elsewhere)

		Girls		*Boys*	
		Geo	*Alg*	*Geo*	*Alg*
Girls	*Geo*	14.46			
	Alg	$r = .19$	16..64		
Boys	*Geo*			17.03	
	Alg			.26	18.69

The full tabulation of the random part of Model 2 is:

PARAMETER	ESTIMATE	S. ERROR	CORR.
School			
σ_v^2(geo_girl)	1.236	0.4214	1
σ_v^2(alg_girl)	0.7531	0.3323	1
σ_v(geo_boy, geo_girl)	1.225	0.3864	0.80
s_v(geo_boy)	1.901	0.5651	1
σ_v^2(alg_boy)	0.8258	0.3722	1
Student			
σ_u^2(geo_girl)	16.46	0.7779	1
σ_u(alg_girl, geo_girl)	3.233	0.5589	0.195
σ_u^2(alg_girl)	16.64	0.7872	1
σ_u^2(geo_boy)	17.03	0.8153	1
σ_u(alg_boy, geo_boy)	4.637	0.6196	0.26
σ_u^2(alg_boy)	18.69	0.8936	1

School Residual Ranks from Model 2

School	geo_girl	alg_girl	alg_boy	geo_boy
1	31	30	.	.
2	37	37	47	35
3	3	7	7	3
4	8	33	22	6
5	28	46	.	.
6	21	26	14	21
7	19	41	42	15
8	27	25	40	14
9	58	1	16	54
10	11	40	6	20
11	48	4	15	34
12	43	35	54	25
13	13	52	4	36
14	53	2	26	30
15	25	45	8	38
16	29	12	1	31
17	39	53	50	41
18	9	13	10	7
20	57	3	51	46
21	41	32	36	37
22	45	31	28	43
23	6	11	12	4
24	15	54	25	23
25	33	20	43	22
26	18	51	18	29
27	42	34	13	49
28	44	39	38	47
29	2	57	46	2
30	50	5	.	.
31	23	36	.	.
32	24	17	48	8
33	40	10	33	24
34	55	14	17	53
35	12	42	49	13
36	32	9	3	32
37	52	24	20	51
39	30	19	41	16
40	36	18	39	28
41	17	6	11	11
42	22	8	2	19
43	4	49	5	18
44	20	16	34	12
45	46	21	30	42
46	34	23	27	26
50	1	56	37	1
51	51	28	29	48
52	16	58	44	33
53	14	50	52	10
54	7	15	9	9
55	49	29	35	39
56	26	38	19	27
57	56	27	53	50
58	35	44	32	40
59	38	47	21	44
60	54	22	24	52
61	10	55	23	17
62	47	43	31	45
64	5	48	45	5

Schools ranked according to their residuals for girls geometry (Model 2)

School	geo_girl	alg_girl	alg_boy	geo_boy
9	58	1	16	54
20	57	3	51	46
57	56	27	53	50
34	55	14	17	53
60	54	22	24	52
14	53	2	26	30
37	52	24	20	51
51	51	28	29	48
30	50	5	.	.
55	49	29	35	39
11	48	4	15	34
62	47	43	31	45
45	46	21	30	42
22	45	31	28	43
28	44	39	38	47
12	43	35	54	25
27	42	34	13	49
21	41	32	36	37
33	40	10	33	24
17	39	53	50	41
59	38	47	21	44
2	37	37	47	35
40	36	18	39	28
58	35	44	32	40
46	34	23	27	26
25	33	20	43	22
36	32	9	3	32
1	31	30	.	.
39	30	19	41	16
16	29	12	1	31
5	28	46	.	.
8	27	25	40	14
56	26	38	19	27
15	25	45	8	38
32	24	17	48	8
31	23	36	.	.
42	22	8	2	19
6	21	26	14	21
44	20	16	34	12
7	19	41	42	15
26	18	51	18	29
41	17	6	11	11
52	16	58	44	33
24	15	54	25	23
53	14	50	52	10
13	13	52	4	36
35	12	42	49	13
10	11	40	6	20
61	10	55	23	17
18	9	13	10	7
4	8	33	22	6
54	7	15	9	9
23	6	11	12	4
64	5	48	45	5
43	4	49	5	18
3	3	7	7	3
29	2	57	46	2
50	1	56	37	1

Schools ranked according to their residuals for girls algebra (Model 2)

School	geo_girl	alg_girl	alg_boy	geo_boy
52	16	58	44	33
29	2	57	46	2
50	1	56	37	1
61	10	55	23	17
24	15	54	25	23
17	39	53	50	41
13	13	52	4	36
26	18	51	18	29
53	14	50	52	10
43	4	49	5	18
64	5	48	45	5
59	38	47	21	44
5	28	46	.	.
15	25	45	8	38
58	35	44	32	40
62	47	43	31	45
35	12	42	49	13
7	19	41	42	15
10	11	40	6	20
28	44	39	38	47
56	26	38	19	27
2	37	37	47	35
31	23	36	.	.
12	43	35	54	25
27	42	34	13	49
4	8	33	22	6
21	41	32	36	37
22	45	31	28	43
1	31	30	.	.
55	49	29	35	39
51	51	28	29	48
57	56	27	53	50
6	21	26	14	21
8	27	25	40	14
37	52	24	20	51
46	34	23	27	26
60	54	22	24	52
45	46	21	30	42
25	33	20	43	22
39	30	19	41	16
40	36	18	39	28
32	24	17	48	8
44	20	16	34	12
54	7	15	9	9
34	55	14	17	53
18	9	13	10	7
16	29	12	1	31
23	6	11	12	4
33	40	10	33	24
36	32	9	3	32
42	22	8	2	19
3	3	7	7	3
41	17	6	11	11
30	50	5	.	.
11	48	4	15	34
20	57	3	51	46
14	53	2	26	30
9	58	1	16	54

School Outliers according to geometry

NAME	COL1	COL2	COL3	COL4	COL5
Above the mean for girls geometry	9	20	57	34	14
Above the mean for boys geometry	9	34	57	.	.
Below the mean for girls geometry	3	29	50	.	.
Below the mean for boys geometry	23	3	29	50	.

School Outliers according to algebra

NAME	COL1	COL2	COL3
Above the mean for girls algebra	52	.	.
Above the mean for boys algebra	12	.	.
Below the mean for girls algebra	20	14	9
Below the mean for boys algebra	.	.	.

School Outliers according to difference between geometry and algebra

NAME	COL1	COL2	COL3	COL4	COL5	COL6	COL7
Large differences between rankings on girls geometry and algebra	9	20	14	11	52	29	50
Large differences between rankings on boys geometry and algebra	34	50	29	9	.	.	.

Largest changes in rankings from model 1 to model 2

School ID	Change in geo_girl	School ID	Change in alg_girl	School ID	Change in geo_boy	School ID	Change in alg_boy
13	-13	2	-19	33	-10	4	-18
26	-13	34	-18	25	-8	24	-17
1	-12	28	-15	26	-7	59	-17
32	14	50	20	40	6	40	14
36	19	62	29	36	9	25	17

The next two models are included to illustrate what happens when the random effect at school level is removed from gender and attached instead to the intercept term, which is common to both boys and girls. This is equivalent to pooling schools' performances for their boys and their girls, in other words assuming their 'effects' are the same for either gender. The fixed part changes very little, but a slightly different ranking of schools arises.

5.5 Model 3

Model 3 (as Model 1, but with no random effect of gender at school level)

The model for the fixed part is:

$$\text{predicted geometry score} = 4.26 + 0.11 KS3test + 0.84 girl\ ,$$
$$\text{predicted algebra score} = 3.24 + 0.13 KS3test + .97\, girl\,,$$

parameter	estimate	s. error(u)
alg_cons	3.244	0.4375
alg_girl	0.9673	0.2006
alg_KS3test	0.135	0.004693
geo_cons	4.258	0.4424
geo_girl	0.8442	0.1969
geo_KS3test	0.1127	0.004664

The estimated residual variance/correlation matrix at school level is

	Geo	*Alg*
Geo	1.484	
Alg	$r = .10$	1.076

,

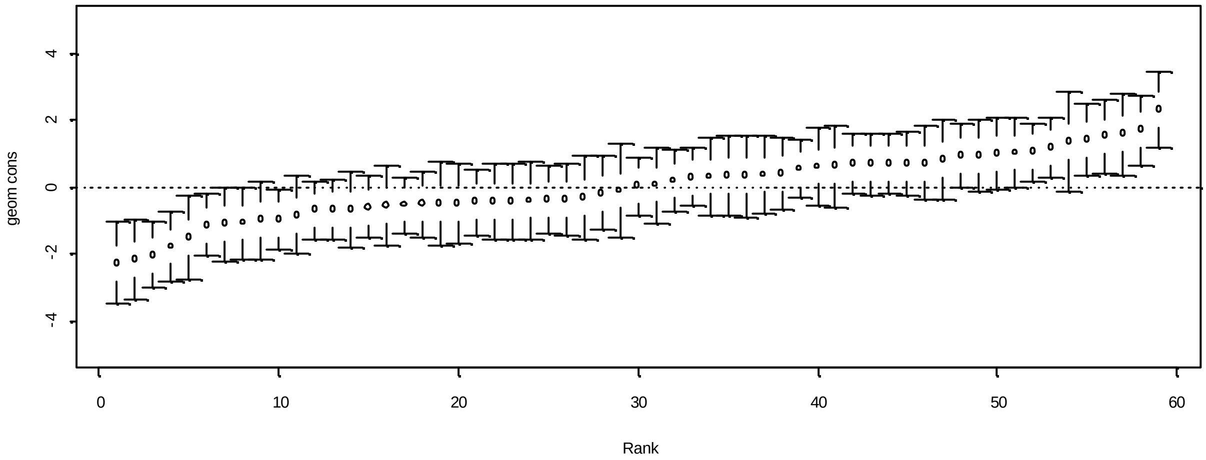
which compares in an obvious way with that for Model 1. The residual variance/correlation matrix at student level is almost unchanged from Model 1.

Student level (variances on the diagonal; correlations elsewhere)

		Girls		*Boys*	
		Geo	*Alg*	*Geo*	*Alg*
Girls	*Geo*	16.55			
	Alg	$r = .19$	16..70		
Boys	*Geo*			17.25	
	Alg			.25	18.67

The full tabulation of the random part follows:

PARAMETER	ESTIMATE	S. ERROR	CORR.
School			
$\sigma_v^2(\text{geo\_cons})$	1.076	0.3095	1
$\sigma_v(\text{geo\_cons, alg\_cons})$	0.1264	0.2455	0.100
$\sigma_v^2(\text{alg\_cons})$	1.484	0.3824	1
Student			
$\sigma_u^2(\text{geo\_girl})$	16.55	0.7746	1
$\sigma_u(\text{alg\_girl, geo\_girl})$	3.163	0.5604	0.190
$\sigma_u^2(\text{alg\_girl})$	16.7	0.7819	1
$\sigma_u^2(\text{geo\_boy})$	17.25	0.8176	1
$\sigma_u(\text{alg\_boy, geo\_boy})$	4.538	0.6197	0.253
$\sigma_u^2(\text{alg\_boy})$	18.67	0.8829	1



4
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geom cons
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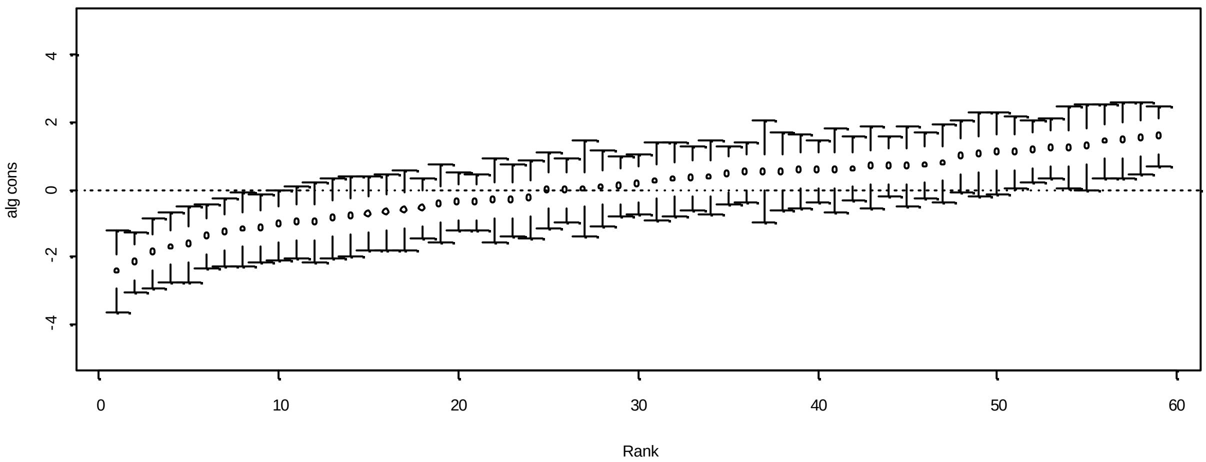
60
Rank



4
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0
-2
-4
alg cons
0
10
20
30
40
50
60
Rank

School Residual Ranks from Model 3

School	Geometry	Algebra
1	43	32
2	41	57
3	5	6
4	4	30
5	30	41
6	24	13
7	22	50
8	17	51
9	58	8
10	16	42
11	31	4
12	34	45
13	32	22
14	39	5
15	25	14
16	19	1
17	47	46
18	6	9
20	55	34
21	48	38
22	44	31
23	3	10
24	23	52
25	36	17
26	35	39
27	53	33
28	52	53
29	1	48
30	51	20
31	29	27
32	8	28
33	40	11
34	57	24
35	14	54
36	13	2
37	49	19
39	21	21
40	28	23
41	18	3
42	11	7
43	12	18
44	15	16
45	46	15
46	26	29
50	2	35
51	50	37
52	33	58
53	20	55
54	9	12
55	45	43
56	27	26
57	54	47
58	38	49
59	37	44
60	56	36
61	10	40
62	42	25
64	7	56

Schools ranked according to their residuals for girls geometry (Model 3)

School	Geometry	Algebra
9	58	8
34	57	24
60	56	36
20	55	34
57	54	47
27	53	33
28	52	53
30	51	20
51	50	37
37	49	19
21	48	38
17	47	46
45	46	15
55	45	43
22	44	31
1	43	32
62	42	25
2	41	57
33	40	11
14	39	5
58	38	49
59	37	44
25	36	17
26	35	39
12	34	45
52	33	58
13	32	22
11	31	4
5	30	41
31	29	27
40	28	23
56	27	26
46	26	29
15	25	14
6	24	13
24	23	52
7	22	50
39	21	21
53	20	55
16	19	1
41	18	3
8	17	51
10	16	42
44	15	16
35	14	54
36	13	2
43	12	18
42	11	7
61	10	40
54	9	12
32	8	28
64	7	56
18	6	9
3	5	6
4	4	30
23	3	10
50	2	35
29	1	48

Schools ranked according to their residuals for girls algebra (Model 3)

School	Geometry	Algebra
52	33	58
2	41	57
64	7	56
53	20	55
35	14	54
28	52	53
24	23	52
8	17	51
7	22	50
58	38	49
29	1	48
57	54	47
17	47	46
12	34	45
59	37	44
55	45	43
10	16	42
5	30	41
61	10	40
26	35	39
21	48	38
51	50	37
60	56	36
50	2	35
20	55	34
27	53	33
1	43	32
22	44	31
4	4	30
46	26	29
32	8	28
31	29	27
56	27	26
62	42	25
34	57	24
40	28	23
13	32	22
39	21	21
30	51	20
37	49	19
43	12	18
25	36	17
44	15	16
45	46	15
15	25	14
6	24	13
54	9	12
33	40	11
23	3	10
18	6	9
9	58	8
42	11	7
3	5	6
14	39	5
11	31	4
41	18	3
36	13	2
16	19	1

School Outliers according to geometry

NAME	COL1	COL2	COL3	COL4	COL5
Above the mean for geometry	9	34	20	57	.
Below the mean for geometry	3	4	23	50	29

School Outliers according to algebra

NAME	COL1	COL2	COL3	COL4	COL5	COL6	COL7
Above the mean for algebra	52	2	.	.	.	.	.
Below the mean for algebra	42	3	14	11	41	36	16

School Outliers according to difference between geometry and algebra

NAME	COL1	COL2	COL3	COL4	COL5	COL6	COL7	COL8
Large differences between rankings on geometry and algebra	9	14	34	35	52	64	50	29

5.6 Model 4

Model 4 (as Model 2 but with no random effect of gender at school level)

The model for the fixed part is:

$$\text{predicted geometry score} = 3.42 + 0.11 KS3test + 0.82 girl + 0.02 s\%A\_C,$$
$$\text{predicted algebra score} = 1.72 + 0.13 KS3test + 0.98 girl + 0.27 tage + 1.13 tHE + 0.92 stext,$$

similar to Model 2. Standard errors are as in the table below:

parameter	estimate	s. error(u)
alg_cons	1.721	0.6342
alg_girl	0.983	0.1998
alg_KS3test	0.1337	0.004658
alg_tage	0.2682	0.1112
alg_tHE	1.128	0.4322
alg_stext	0.9224	0.3221
geo_cons	3.421	0.6626
geo_girl	0.8286	0.1969
geo_KS3test	0.1105	0.004813
geo_s%A_C	0.01973	0.01164

The estimated residual variance/correlation matrix at school level is:

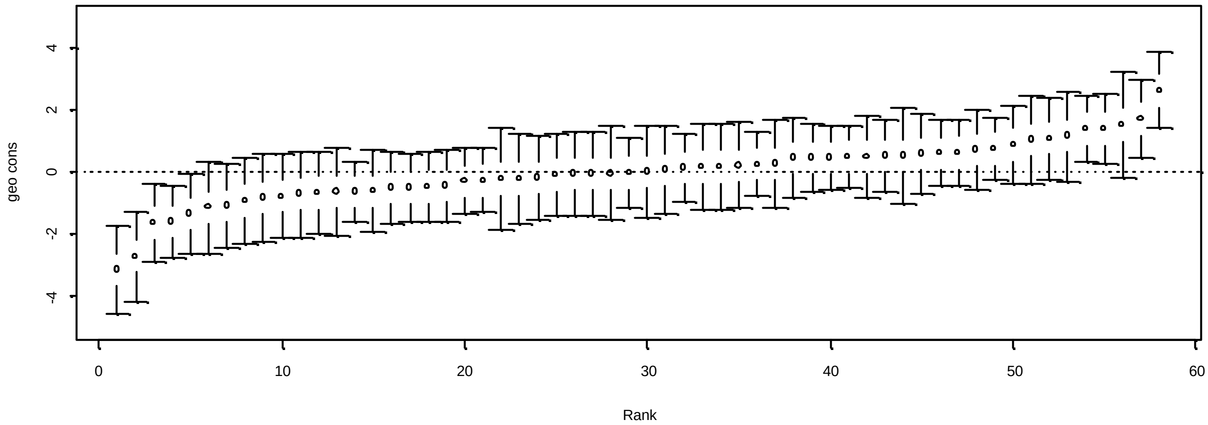
$$\begin{array}{c|cc} & G & A \\ \hline G & 1.395 & \\ A & r=-.048 & .87 \end{array}.$$

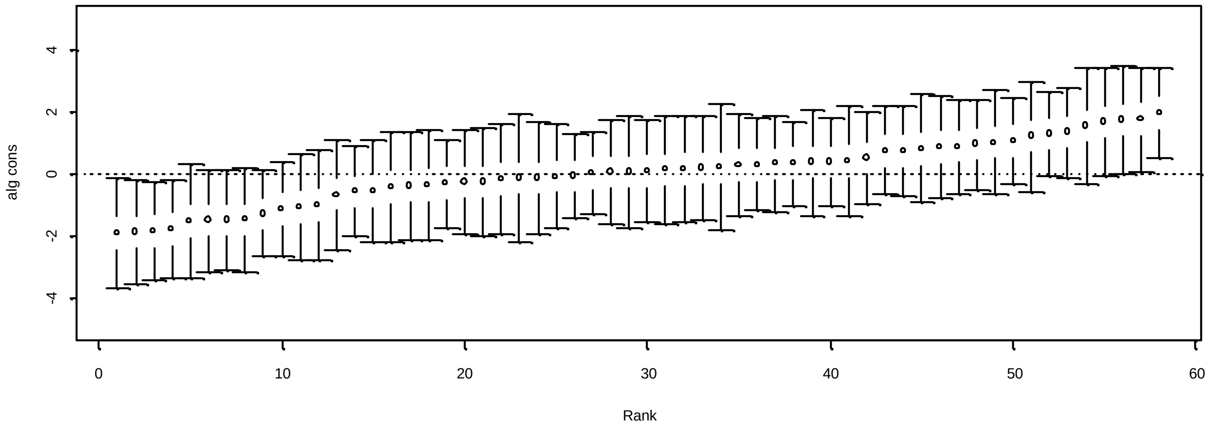
Student level (variances on the diagonal; correlations elsewhere)

		Girls *Geo*	*Girls* *Alg*	*Boys* *Geo*	*Boys* *Alg*
Girls	*Geo*	16.54			
	Alg	$r = .19$	16.55		
Boys	*Geo*			18.6	
	Alg			.25	21.93

Compared to Model 2, it is relatively straightforward to detect correlation at school level between performance in Geometry and in Algebra. Statistical test shows that there is no difference in variance between boys' Algebra and girls'. The full tabulation of the random part is:

PARAMETER	ESTIMATE	S. ERROR	CORR.
School			
$\sigma_v^2(\text{geo\_cons})$	1.395	0.3654	1
$\sigma_v(\text{geo\_cons, alg\_cons})$	-0.05359	0.2225	-0.0485
$\sigma_v^2(\text{alg\_cons})$	0.8766	0.2705	1
Student			
$\sigma_u^2(\text{geo\_girl})$	16.54	0.7736	1
$\sigma_u(\text{alg\_girl, geo\_girl})$	3.145	0.5573	0.190
$\sigma_u^2(\text{alg\_girl})$	16.55	0.7744	1
$\sigma_u^2(\text{geo\_boy})$	18.6	0.8789	1
$\sigma_u(\text{alg\_boy, geo\_boy})$	4.55	0.6185	0.254
$\sigma_u^2(\text{alg\_boy})$	21.93	1.036	1


geo cons
4
2
0
-2
-4
0
10
20
30
40
50
60
Rank


alg cons
4
2
0
-2
-4
0
10
20
30
40
50
60
Rank

School Residual Ranks from Model 4

School	Geometry	Algebra
1	33	28
2	41	50
3	3	6
4	5	25
5	34	49
6	21	14
7	17	48
8	19	37
9	58	4
10	15	13
11	38	7
12	36	52
13	27	20
14	47	9
15	31	21
16	26	2
17	45	57
18	7	11
20	54	19
21	43	42
22	48	33
23	4	10
24	20	43
25	29	36
26	25	35
27	51	22
28	50	46
29	2	55
30	44	5
31	24	39
32	13	41
33	35	17
34	57	15
35	18	53
36	28	1
37	53	18
39	22	34
40	37	31
41	12	8
42	16	3
43	9	16
44	14	27
45	49	26
46	30	24
50	1	51
51	52	30
52	32	58
53	11	56
54	8	12
55	42	32
56	23	29
57	55	47
58	39	44
59	40	38
60	56	23
61	10	45
62	46	40
64	6	54

Schools ranked according to their residuals for girls geometry (Model 4)

School	Geometry	Algebra
9	58	4
34	57	15
60	56	23
57	55	47
20	54	19
37	53	18
51	52	30
27	51	22
28	50	46
45	49	26
22	48	33
14	47	9
62	46	40
17	45	57
30	44	5
21	43	42
55	42	32
2	41	50
59	40	38
58	39	44
11	38	7
40	37	31
12	36	52
33	35	17
5	34	49
1	33	28
52	32	58
15	31	21
46	30	24
25	29	36
36	28	1
13	27	20
16	26	2
26	25	35
31	24	39
56	23	29
39	22	34
6	21	14
24	20	43
8	19	37
35	18	53
7	17	48
42	16	3
10	15	13
44	14	27
32	13	41
41	12	8
53	11	56
61	10	45
43	9	16
54	8	12
18	7	11
64	6	54
4	5	25
23	4	10
3	3	6
29	2	55
50	1	51

Schools ranked according to their residuals for girls algebra (Model 4)

School	Geometry	Algebra
52	32	58
17	45	57
53	11	56
29	2	55
64	6	54
35	18	53
12	36	52
50	1	51
2	41	50
5	34	49
7	17	48
57	55	47
28	50	46
61	10	45
58	39	44
24	20	43
21	43	42
32	13	41
62	46	40
31	24	39
59	40	38
8	19	37
25	29	36
26	25	35
39	22	34
22	48	33
55	42	32
40	37	31
51	52	30
56	23	29
1	33	28
44	14	27
45	49	26
4	5	25
46	30	24
60	56	23
27	51	22
15	31	21
13	27	20
20	54	19
37	53	18
33	35	17
43	9	16
34	57	15
6	21	14
10	15	13
54	8	12
18	7	11
23	4	10
14	47	9
41	12	8
11	38	7
3	3	6
30	44	5
9	58	4
42	16	3
16	26	2
36	28	1

School Outliers according to geometry

NAME	COL1	COL2	COL3	COL4	COL5
Above the mean for geometry	9	34	57	20	.
Below the mean for geometry	4	23	3	29	50

School Outliers according to algebra

NAME	COL1	COL2	COL3	COL4
Above the mean for algebra	52	17	.	.
Below the mean for algebra	9	42	16	36

School Outliers according to difference between geometry and algebra

NAME	COL1	COL2	COL3	COL4	COL5	COL6	COL7	COL8
Large differences between rankings on geometry and algebra	9	34	14	20	53	64	50	29

Largest changes in rankings from model 3 to model 4

School ID	Change in geometry	School ID	Change in algebra
1	-10	10	-29
26	-10	30	-15
53	-9	20	-15
40	9	50	16
36	15	25	19

6 Summary

In this report, we have assessed the total scores for year 9 constructive proof in Geometry and Algebra (combined with Logic) with a special interest in the variance changes at school, class and student levels with introducing a number of characteristics at school level, class level and student level and identifying school outliers.

We used three important variables: year 8 proof score, baseline math test score and Key Stage 3 test score as the baseline information in the data analysis, with an aim to compare the results when different baseline information was used in the analysis. To avoid the arbitrariness of coding covariates in the multivariate analyses, we have used ACE algorithm to transform the independent and dependent variables.

The analyses show a quite comparable results on the fix effects of independent variables on proof score when three baseline information variables were used. For example, the percentage of the school GCSE pass rate at A*-C) was found statistically significantly associated with the geometry score whether baseline information was year 8 proof score, baseline math test score or Key Stage 3 test score. Also, it was found that three additional variables are found to be significant predictors of proof algebra score. Students taught by a teacher with a MSc or PhD degree education had a higher proof score than their counterparts, and students using of textbooks other than "Key Math" had a higher proof score than those using other textbooks.

In terms of the random effects, it was found that there is statistically significant variation at school level and class level. However, class variation is no longer significant when baseline information is introduced into the model. When the statistical significant variables are selected and introduced into the most parsimonious model, high correlation is found between boys and girls geometry scores. In addition, including the additional predictors in the model reduces the school-level variation. We also estimated two models to illustrate what happens when the random effect at school level is removed from gender and attached instead to the intercept term, which is common to both boys and girls. This is equivalent to pooling schools' performances for their boys and their girls, in other words assuming their 'effects' are the same for either gender. The fixed part changes very little, but a slightly different ranking of schools arises.

In terms of school outliers, different rankings are generated when three different baseline information was used. There are two main reasons for this. First, analyses corresponding to the three baseline variables are based on different sample sizes as show on page 1: 3909, 3859 and 3743 students for models with year 8 proof score, baseline math test score and Key Stage 3 test score as baseline information, respectively. Second, variances at the school level are reduced at different scale when three different baseline information was controlled for. The following table compares the school level variance when different baseline information is used in Model 2. It is obvious that Key Stage 3 test score reduced very large part of school level variance of algebra scores for both girls and boys.

Comparison of school level variance changes when different baseline information is used

Variance	Year 8 Proof Score	Year 8 Baseline Test Score	Key Stage 3 Test Score
$\sigma_v^2(\text{geo\_girl})$	1.046	1.752	1.236
$\sigma_v^2(\text{alg\_girl})$	2.73	2.166	0.753
$s_v(\text{geo\_boy})$	2.538	2.486	1.901
$\sigma_v^2(\text{alg\_boy})$	1.871	1.992	0.8258

References

Breiman, L., and J.H. Friedman (1985). Estimating Optimal Transformations for Multiple Regression and Correlation, *Journal of American Statistical Association*, 80:580-597.

De Veaux, R.D. (1989) Finding Transformation for Regression Using the ACE Algorithm. *Sociological Methods and Research*, 18:327-359.

Goldstein, H. (1995) *Multilevel Statistical Models*, 2Ed (London: Edward Arnold)

Healy, L. and Hoyles, C. (1999) *Justifying and Proving in School Mathematics: Technical report on the nationwide survey* (London: Institute of Education, June)

Raftery, A. E. (1995). Bayesian Model Selection in Social Research (with Discussion by A. Gelman & D. B. Robin, and R. M. Hauser, and a Rejoinder). *Sociological Methodology*, 25:111-195.

Venables, W. N. and B. D. Ripley. 1994. *Modern Applied Statistics with S-Plus.* New York: Springer-Verlag Inc.

Wang D, Murphy M. Covariates of infant mortality: an exploratory approach. *Social Biology*, 1998: **45**:21-38.