1 Introduction

The Proof Materials Project ran for two years, from January 2004 to December 2005. Its stated aim was

> to produce guidance and stimulus materials for teachers that can be used to develop the mathematical reasoning of pupils across the ability range in Key Stages 3 and 4.

To achieve this, we worked with two groups of teachers, one in north east Hampshire, the other in Bury St Edmunds, Suffolk, who trialled materials in their classrooms. Each group met regularly* to share and reflect on their experiences, and to plan the next classroom activities. On occasions, teachers also visited each other in their schools.

The meetings took place during the school day, rather than in twilight sessions, and we are extremely grateful for the support given by Wendy Hoskin, county inspector/advisor-mathematics, Hampshire, and Colin Matthews, former mathematics advisor, Suffolk, who, together with the generosity of their local authorities, made this possible.

The project followed on from our Longitudinal Proof Project (1999 - 2003). In that project, we used written tests to survey the proof strategies used by high attaining secondary school students. The students were tested towards the end of Year 8 and again in Year 9 and Year 10. (Altogether, 1512 students took all three tests.) We found that students had a tendency to use empirical arguments and, in common with many other studies, one could summarise the findings as 'students are not very good at producing proof arguments' and 'proof is difficult'.

However, the research data can be viewed far more positively. For a start, it should be borne in mind that most of our students would not have devoted much time to proof activities, as this was not emphasised in the then current mathematics curriculum for English schools (the 2008 revised National Curriculum hopes to change that....); also, our students' responses were produced cold, in a written test that was not connected to their day to day work in the classroom.

Some of the students *did* produce structural arguments, ie arguments based on mathematical relationships rather than just on empirical data. Consider the statement below, from the Year 10 proof test:

> **When you add any 2 odd numbers, your answer is always even.**

Students were asked to prove whether the statement was true or false. The most common response (from 31 % of our sample of 1512 students) was just to give an empirical 'proof', ie just to list examples that supported the statement (such as '3 + 5 = 8'). However, a similar proportion (29 % of the sample) attempted some kind of structural proof based on the properties of odd and even numbers (eg that an odd number can be partitioned into two equal amounts, with one left over)**.

This provides an 'existence proof' that there *are* students who have some awareness of the importance of structure in developing mathematical arguments. It also raises the possibility that if teachers focus students' attention on structure then there might be many more students who can discern structure and use it, and not just those in 'high attaining' classes as in our sample***. It was this hypothesis (or belief) that drove the new project, and I hope that the following chapters will show that this was justified.

The teachers in both of our groups varied considerably in terms of age and experience. Some were heads of mathematics, others had only one or two years' experience of teaching. Some of them knew each other well and all seemed used to working collaboratively. The teachers were highly skilled and it was a pleasure and privilege to be in their classrooms.

Some of the teachers were familiar with the ideas and ways of working that I wanted to explore - in particular, the emphasis on looking for structure - but others were not. For the latter teachers, it often took a long time to feel at home with these ideas, and a lot longer than their rhetoric might have suggested. This surprised me initially, given their knowledge and skills, and their enthusiasm for the project, but it brought home the difficulty of changing one's practice (as opposed to changing one's way of talking about one's practice!). However, all the teachers were very willing (and kind enough) to try out ideas, even if this meant making a change from their usual way of teaching and taking risks. Such trials were often very illuminating.

---

* From the summer term 2004 to the summer term 2005, the Hampshire group met two or three times per term, sometimes for the whole day, and the Suffolk group met at least once per term.

** A further 19 % gave a proof based on the fact that (in base 10) odd numbers end in 1, 3, 5, 7 or 9 and that the sum of any two such digits ends in 0, 2, 4, 6 or 8, which characterises even numbers. Though this is an empirical argument, it is also exhaustive, ie it applies to all odd numbers. [The test item was preceded by a multiple choice question in which various 'proofs' were given for a statement about the sum of even numbers, which included this kind of exhaustive proof. We discuss this further in chapter 2 (page 2.2).]

*** We are not alone in thinking this might be true. For example, Johnston-Wilder and Mason (2005) write that "children demonstrate from a very young age that they have natural powers of locating relationships..." (in *Developing Thinking in Geometry*, page 240). Similarly, Blanton suggests that even with elementary grade students it is possible to "cultivate habits of mind that attend to the deeper, underlying structure of mathematics" (in Katz (2007), *Algebra: Gateway to a Technological Future*, page 7).

However, they usually only ran over one or two lessons and, as such, were not extensive enough to provide evidence of long term change, be it in the students or the teachers - although all the teachers felt that over the course of the project they had developed new insights and ways of working.

I hope that the evidence presented in the chapters that follow will throw light on the challenges involved in developing students' proof arguments and also provide examples of the fascinating and powerful ways of thinking that students demonstrated - whether or not some of these represent permanent changes in their thinking, ie new habits of mind.

Acknowledgements I would like to thank all the following people for the work they put into the project, for their enthusiasm and for the many kindnesses that were shown to me. Thanks also to the students who took part in the project, as well as to the headteachers of the participating schools for the hospitality I received on my many visits.

Bury St Edmunds

Teachers:

Chris Burrell
Katie Cobb
Gill Turner
Sarah Whyand
Chris Dale (numeracy consultant)
Colin Matthews (mathematics advisor)

Schools:

Howard Middle School
King Edward VI Upper School
St Louis Middle School
St Benedict's School

Hampshire

Teachers:

Kim Cappleman
Marian Flaxman
David Gillon
Sheila Gray
Helen Humble
Neil Howard
Emma Lipscombe-Holmes
John Reid
Margaret Schofield
Karen-Anne Spencer
Heidi Whitney
Nigel Wills
Bryan Winter
Wendy Hoskin (county inspector/advisor-mathematics)
Tessa Ingrey (numeracy consultant)

Schools:

Amery Hill School
Bishop Challoner Secondary School
Perins Community School
The Connaught School
Court Moor School
Fort Hill Community School
Henry Beaufort School
The Hurst Community College
Robert May's School