Appendix A Y9 Proof Survey

INSTITUTE OF EDUCATION
UNIVERSITY OF LONDON

Year 9 Proof Survey

Name .... Maths Class .....
first name surname

School ..... Boy or Girl .....

Date of Birth .... Today's Date ....
day month year day month year

You have 55 minutes to answer these questions.

In two of the questions you will be asked to choose from a range of answers.

In all the other questions, you will be asked to produce your own answers. We are interested in your thinking as well as your answers, so please show all your rough working for these questions.

Put your rough working on the same page as your answer; use the answer box or any spare space on the page.

In most questions you will be asked for explanations. Make these as clear as you can, but don't make them longer than necessary.

Use a pen. You may cross things out, but do not rub out any of your work and do not use correction fluid.

Do not use a calculator.

You might find some of the questions quite difficult. Don't worry. If you get stuck on a question, leave it till later.

On the last page there is a questionnaire. Only fill this in if you have done all you can on the other questions and there is time left over.

A

Sch

Cla

Stu

Longitudinal Proof Project

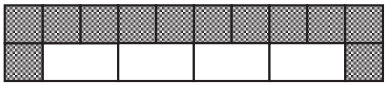
Funded by the Economic and Social Research Council

A1 Larry has some white rectangular tiles and some grey square tiles.
The white tiles are twice as long as the grey tiles but have the same width.

Please leave blank

He makes a row of white tiles, like this:

He then builds a 'bridge' of grey tiles over the white tiles, like this:



a) How many grey tiles does he need to build a bridge over a row of 40 white tiles? . . . . . .

Show how you obtained your answer.

b) Write an expression for the number of grey tiles needed for a row of *n* white tiles. . . . . .

G1 Tim sketches a quadrilateral. He draws the diagonals of the quadrilateral.

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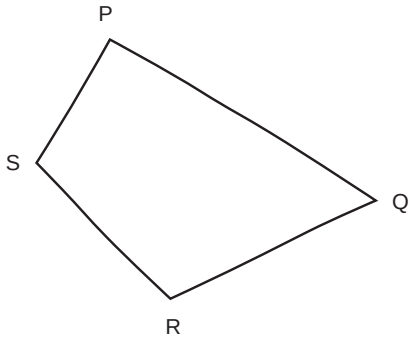
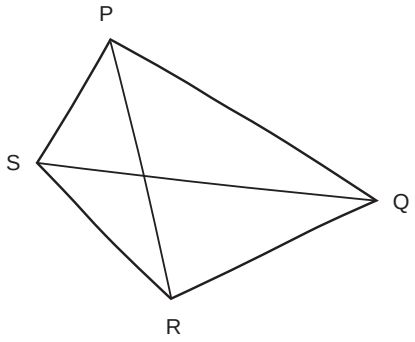




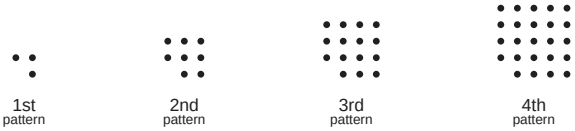
Tim notices that one of the diagonals has cut the area of the quadrilateral in half.

He says
"Whatever quadrilateral I draw,
at least one of the diagonals will always cut the area of the quadrilateral in half".

Is Tim right? . . . . . .

Explain your answer.

A2 Karen and Josie are looking at these first four patterns in a sequence of dot patterns:



Please leave blank

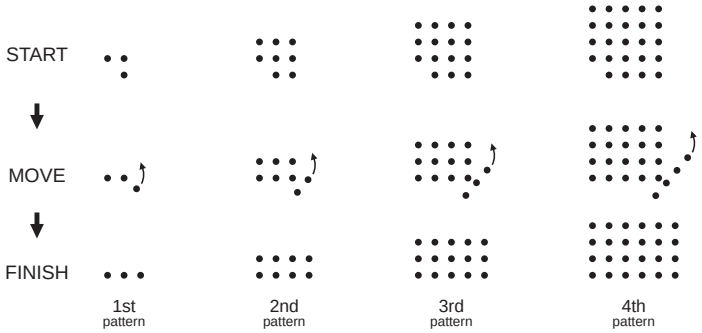
a) Karen wants to calculate the number of dots in the 4th and 20th pattern.
She says each pattern looks like a square with a dot missing from one corner.

Use **Karen's** idea to *calculate* the number of dots in

i. the 4th pattern . . . . . ii. the 20th pattern. . . . . .

Show how you obtained your answer	Show how you obtained your answer

b) Josie also wants to calculate the number of dots in the 4th and 20th pattern.
She says she can change each pattern into a rectangle, like this:



Use **Josie's** idea to *calculate* the number of dots in

i. the 4th pattern . . . . . ii. the 20th pattern. . . . . .

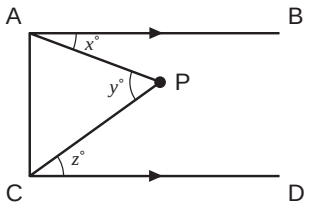
Show how you obtained your answer	Show how you obtained your answer

c) Write an expression for the number of dots in the *n*th pattern, using

i. Karen's way of looking at the pattern . . . . . .

ii. Josie's way of looking at the pattern. . . . . .

G3 In the diagram, line AB is parallel to line CD, and AC is at right angles to both lines.



Points A, B, C and D are fixed.
Point P can move anywhere between AB and CD, but stays connected to A and C
(the straight lines PA and PC can stretch or shrink).

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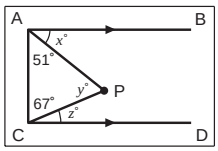
Astrid, Burt, Cleo, Dilip and Emma are discussing whether this statement is true:

***x*° + *z*° is equal to *y*°.**

Astrid's answer



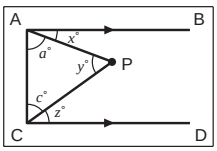
I could have a triangle APC with these angles. →

Then
$y = 180 - 51 - 67 = 62$,
$x = 90 - 51 = 39$, and $z = 90 - 67 = 23$.

But $62 = 39 + 23$, and as
$180 - 51 - 67 = (90 - 51) + (90 - 67)$,
I could have a triangle with other angles.
So $y = x + z$.

So Astrid says it's true

Burt's answer



The angle sum of triangle ACP is 180°, so $y + a + c = 180$.

Angles A and C are 90°, so I can write $90 - x$ for a, and $90 - z$ for c.

So $y + (90 - x) + (90 - z) = 180$,
so $y - x - z = 0$,
so $y = x + z$.

So Burt says it's true

Cleo's answer

I measured the angles in the original diagram. I then moved P to another place and measured the angles again.

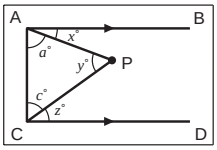
I made this table:

x	z	y
21	36	57
17	32	49

So both times I found that $x + z$ equals y.

So Cleo says it's true

Dilip's answer



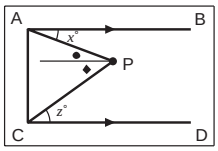
The angle sum of triangle ACP is 180°.

So I can write $a + c = 180 - y = 180 - (x + z)$.
Also $y = 180 - (a + c)$.

So $y = 180 - (180 - (x + z)) = x + z$.

So Dilip says it's true

Emma's answer



I drew a line through P parallel to lines AB and CD.

The new line cuts angle y into two parts.
The top part (●) is equal to x because the new line is parallel to AB. The bottom part (◆) is equal to z because the new line is parallel to CD.
So, altogether, y is equal to $x + z$.

So Emma says it's true

a) Whose answer do you like best? . . . . . .

b) Whose answer is closest to what you would do? . . . . . .

c) Whose answer would get the best mark from your teacher? . . . . . .

G3 *Continued*

Please leave blank

d) For each of the following, circle whether you agree, don't know, or disagree.

The statement is: $x^\circ + z^\circ$ **is equal to** y°.

	agree	don't know	disagree
Astrid's answer ...			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3
Burt's answer ...			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3
Cleo's answer ...			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3
Dilip's answer ...			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3
Emma's answer ...			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3

LA1 Pam and Viv are thinking about the pair of numbers 5 and 9.

Please leave blank

They notice that the SUM $(5 + 9)$ is EVEN.

They notice that the PRODUCT (5×9) is ODD.

Pam says: If the SUM of two whole numbers is EVEN, their PRODUCT is ODD.

Viv says: If the PRODUCT of two whole numbers is ODD, their SUM is EVEN.

a) Are Pam's and Viv's statements saying the same thing? . . . . . .

b) The PRODUCT of two whole numbers is 1247.

Suppose Viv is right.

Which one of these must also be right? Tick (✓) one box.

❑ You can be sure that the SUM of the two numbers is EVEN.

❑ You can be sure that the SUM of the two numbers is ODD.

❑ You can't be sure whether the SUM is ODD or EVEN until you know what the two numbers are.

c) Is Pam's statement true? . . . . . .

Explain your answer.

d) Is Viv's statement true? . . . . . .

Explain your answer.

A4 a) 4! means $4 \times 3 \times 2 \times 1$.
5! means $5 \times 4 \times 3 \times 2 \times 1$.

Please leave blank

Is 5! exactly divisible by 3 ? . . . . . .

Explain your answer.

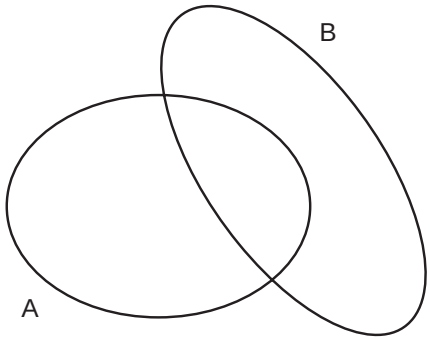
b) What does 50! mean?

c) Is 50! exactly divisible by 19 ? . . . . . .

Explain your answer.

G2 The two ovals A and B have the same area.

The ovals overlap.



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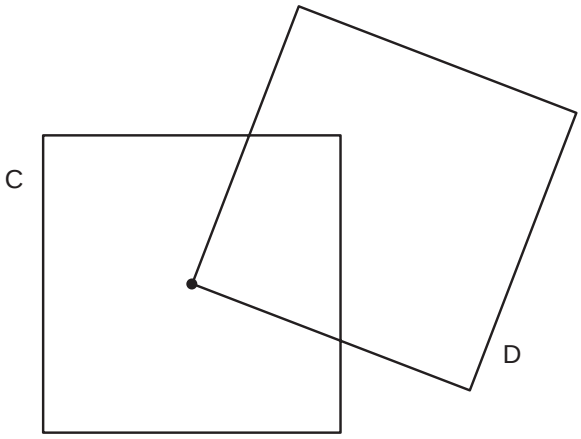
a) Do the two non-overlapping regions have the same area?

. . . . . .

Explain your answer.

b) Squares C and D are identical. One corner of D is at the centre of C.

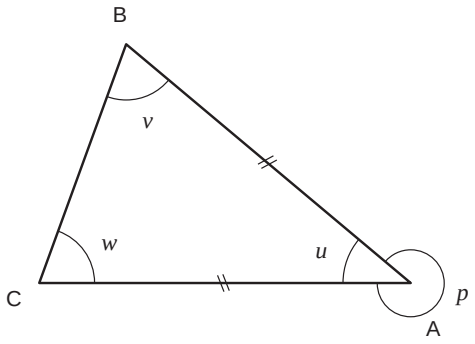
What fraction of C is overlapped by D ?



. . . . . .

Explain your answer.

G4



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The diagram shows a triangle ABC.
Side AB is the same length as side AC.

a) Find the size of angle *v*, when angle *p* is 320˚. . . . . .

Write down each step of your calculation.

b) Write down your first step again and give a reason for the step.

c) Write down your next steps again and give a reason for each one.

A3 The drawing shows the calender for last July.

A square is drawn around nine of the numbers.

The top-left number and bottom-right number in the square are circled.

Ashok, Beryl, Cora, Dave and Ethan are discussing whether this statement is true:

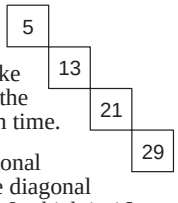
When there are nine numbers in the square, the bottom-right number will be 16 more than the top-left number.

July

M	T	W	T	F	S	S
					1	2
3	4	5	6	7	8	9
10	(11)	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	(27)	28	29	30
31						

Please leave blank

Ashok's answer



Look at a 'diagonal' line like this one. You can see that the numbers increase by 8 each time.

So if you go from one diagonal number to the next-but-one diagonal number, it increases by 8 + 8 which is 16.

So Ashok says it's true

Beryl's answer

For a square with 9 numbers, you can get from the first circled number to the second by going 2 steps across and 2 steps down.

Each step across is an increase of 1 day.
Each step down is an increase of 1 week.

So altogether, the number increases by 1 + 1 + 7 + 7, which is 16.

So Beryl says it's true

Cora's answer

Think of a square of nine numbers.
Call the first number *n*.

Then this shows the first row and column. The numbers go up by a day at a time in each row and by a week at a time in each column.

n	n+1	n+1+1
n+7	-	-
n+7+7	-	-

So the last number is $n + 1 + 1 + 7 + 7$, which is $n + 16$.

So Cora says it's true

Dave's answer

It works for the original square because 27 is 16 more than 11.

(11)	12	13
18	19	20
25	26	(27)

It also works for this square, because 21 is 16 more than 5.

(5)	6	7
12	13	14
19	20	(21)

So Dave says it's true

Ethan's answer

Draw a square full of numbers.
Let x be the first number in the square.
Let d be the number of days in a week.
Let s be the number of numbers in the square.

Then the last number in the square is $x + d + s$.

But $d = 7$ and $s = 9$, so the last number is $x + 7 + 9$ which is $x + 16$.

So Ethan says it's true

a) Whose answer do you like best? . . . . . .

b) Whose answer is closest to what you would do? . . . . . .

c) Whose answer would get the best mark from your teacher? . . . . . .

A3 *Continued*

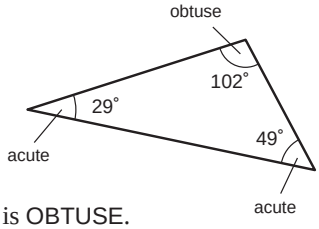
Please leave blank

d) For each of the following, circle whether you agree, don't know, or disagree.

The statement is: **When there are nine numbers in the square, the bottom-right number will be 16 more than the top-left number**.

	agree	don't know	disagree
Ashok's answer ...			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3
Beryl's answer ...			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3
Cora's answer ...			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3
Dave's answer ...			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3
Ethan's answer ...			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3

LG1 Jack and Bob are thinking about the angles of this triangle.



They notice that two angles are ACUTE.

They notice that one angle is OBTUSE.

Jack says: If two angles of a triangle are ACUTE, the third angle is OBTUSE.

Bob says: If one angle of a triangle is OBTUSE, the other two angles are ACUTE.

a) Are Jack's and Bob's statements saying the same thing? . . . . . .

b) A triangle has an OBTUSE angle of 113.62°.

Suppose Bob is right.

Which one of these must also be right? Tick (✓) one box.

- ❑ You can be sure that the other two angles are both ACUTE.
- ❑ You can be sure that the other two angles are not both ACUTE.
- ❑ You can't be sure whether the other two angles are both ACUTE until you know the size of both angles.

c) Is Jack's statement true? . . . . . .

Explain your answer.

d) Is Bob's statement true? . . . . . .

Explain your answer.

Please leave blank

WAIT! *Please go back to any questions you left out, then check all your answers. After that, if there is any time left over, please answer this questionnaire:*

Please leave blank

Z1 a) What did you feel about taking part in this survey?

b) Which question did you like best, and why?

c) Which question did you like least, and why?

d) Please add any other comments, if you wish to, about the survey.

Appendix B Y9 Teacher Questionnaire

INSTITUTE OF EDUCATION
UNIVERSITY OF LONDON

Teacher Questionnaire (Y9)

Name .....

School ..... LEA .....

Name of your Y9 maths class with students involved in the survey .....

Please complete this questionnaire while your students are taking the proof survey.

Complete the details above and on pages 1 and 2, then work through the proof questions that follow.

Sch

Cla

Tea

Longitudinal Proof Project

Funded by the Economic and Social Research Council

Please tick (✓) the appropriate boxes and complete the appropriate blanks

Female ❑ 1 Male ❑ 2

How many years teaching experience did you have at the start of this school year? .....

Your age: under 25 ❑ 1 25 - 29 ❑ 2 30 - 39 ❑ 3 40 - 49 ❑ 4 50 - 59 ❑ 5 60 or more ❑ 6

School responsibility:

Head of maths ❑ Other (please specify) .....

Teaching Qualification:

Please specify type of qualification and subjects studied

Type		Main subject (please specify)	Subsidiary subject (please specify)
Degree (if not BEd)	❑	.....	.....
BEd	❑	.....	.....
PGCE	❑	.....	.....
Cert Ed	❑	.....	.....
Other	❑	.....	.....

Higher Education (apart from above):

Please specify type of qualification and main subject studied

Type		Title (eg MEd)	Main subject	Year completed
Masters	❑	.....	.....	.....
PhD	❑	.....	.....	.....

Continuing Professional Development (CPD) or INSET in mathematics education

a. In this section do NOT include Government INSET for NNS or NOF, or any courses that you have mentioned in the Higher Education section, but DO include activities such as attending courses or conferences, writing text books, serving as an examiner, taking part in projects.

For the previous school year (1999 - 2000), estimate the number of *sessions* you were involved in CDP or INSET in mathematics education (where a session is a morning, afternoon, twilight or evening): .....

b. Current membership of a professional association:

ATM ❑ MA ❑ IMA ❑ Other (please specify) .....

Involvement in extra-curricular mathematics activities with students in your school during 2000 - 2001 (ie activites that are not part of the normal school mathematics curriculum, such as organising a mathematics club, organising students for master classes or UK Maths Challenge, taking students to mathematics events):

Yes ❑ 1 No ❑ 2

Software that you have used this school year with Y9 students:

Logo ❑ Dynamic geometry ❑ Spreadsheet ❑ Database ❑

Graphics calculator ❑ Integrated learning system ❑ SMILE software ❑ The internet ❑

Other (please specify) .....

A3 The drawing shows the calender for last July.

A square is drawn around nine of the numbers.

The top-left number and bottom-right number in the square are circled.

Ashok, Beryl, Cora, Dave and Ethan are discussing whether this statement is true:

When there are nine numbers in the square, the bottom-right number will be 16 more than the top-left number.

July

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					1	2
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10	(11)	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	(27)	28	29	30
31						

a) Please satisfy yourself that the statement above is true, then go on to part b).

b) Consider Ashok, Beryl, Cora, Dave and Ethan's answers on the next page.

i. Give a mark (out of 10) for each answer. A. . . . B. . . . C. . . . D. . . . E. . . .

ii. Whose answer would your students say would get the best mark from you? . . . .

iii. Whose answer is closest to what you would do? . . . .

c) Write a brief comment that might help these two students to move on:

Ashok . . . . . .

. . . . . .

. . . . . .

. . . . . .

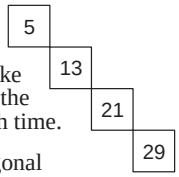
Dave . . . . . .

. . . . . .

. . . . . .

. . . . . .

Ashok's answer



Look at a 'diagonal' line like this one. You can see that the numbers increase by 8 each time.

So if you go from one diagonal number to the next-but-one diagonal number, it increases by 8 + 8 which is 16.

So Ashok says it's true

Beryl's answer

For a square with 9 numbers, you can get from the first circled number to the second by going 2 steps across and 2 steps down.

Each step across is an increase of 1 day.
Each step down is an increase of 1 week.

So altogether, the number increases by 1 + 1 + 7 + 7, which is 16.

So Beryl says it's true

Cora's answer

Think of a square of nine numbers.
Call the first number *n*.

Then this shows the first row and column. The numbers go up by a day at a time in each row and by a week at a time in each column.

n	n+1	n+1+1
n+7	-	-
n+7+7	-	-

So the last number is $n + 1 + 1 + 7 + 7$, which is $n + 16$.

So Cora says it's true

Dave's answer

It works for the original square because 27 is 16 more than 11.

(11)	12	13
18	19	20
25	26	(27)

It also works for this square, because 21 is 16 more than 5.

(5)	6	7
12	13	14
19	20	(21)

So Dave says it's true

Ethan's answer

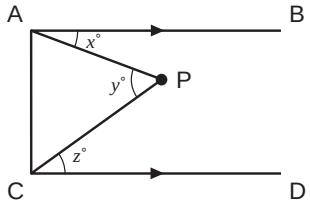
Draw a square full of numbers.
Let x be the first number in the square.
Let d be the number of days in a week.
Let s be the number of numbers in the square.

Then the last number in the square is $x + d + s$.

But $d = 7$ and $s = 9$, so the last number is $x + 7 + 9$ which is $x + 16$.

So Ethan says it's true

G3 In the diagram, line AB is parallel to line CD, and AC is at right angles to both lines.



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Point P can move anywhere between AB and CD, but stays connected to A and C
(the straight lines PA and PC can stretch or shrink).

Astrid, Burt, Cleo, Dilip and Emma are discussing whether this statement is true:

***x*° + *z*° is equal to *y*°.**

a) Please satisfy yourself that the statement above is true, then go on to part b).

b) Consider Astrid, Burt, Cleo, Dilip and Emma's answers on the next page.

i. Give a mark (out of 10) for each answer. A . . . . B . . . . C . . . . D . . . . E . . . .

ii. Whose answer would your students say would get the best mark from you? . . . .

iii. Whose answer is closest to what you would do? . . . .

c) Write a brief comment that might help these two students to move on:

Astrid . . . . . .

. . . . . .

. . . . . .

. . . . . .

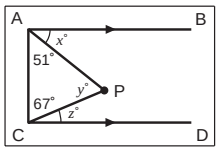
Cleo . . . . . .

. . . . . .

. . . . . .

. . . . . .

Astrid's answer



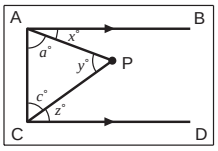
I could have a triangle APC with these angles. →

Then
$y = 180 - 51 - 67 = 62$,
$x = 90 - 51 = 39$, and
$z = 90 - 67 = 23$.

But $62 = 39 + 23$, and
$180 - 51 - 67 = (90 - 51) + (90 - 67)$,
so $y = x + z$.

So Astrid says it's true

Burt's answer



The angle sum of triangle ACP is 180°, so $y + a + c = 180$.

Angles A and C are 90°, so I can write $90 - x$ for a, and $90 - z$ for c.

So $y + (90 - x) + (90 - z) = 180$,
so $y - x - z = 0$,
so $y = x + z$.

So Burt says it's true

Cleo's answer

I measured the angles in the original diagram. I then moved P to another place and measured the angles again.

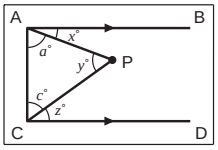
I made this table:

x	z	y
21	36	57
17	32	49

So both times I found that $x + z$ equals y.

So Cleo says it's true

Dilip's answer



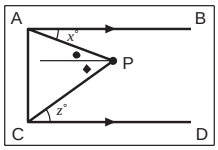
The angle sum of triangle ACP is 180°.

So I can write $a + c = 180 - y = 180 - (x + z)$.
Also $y = 180 - (a + c)$.

So $y = 180 - (180 - (x + z)) = x + z$.

So Dilip says it's true

Emma's answer



I drew a line through P parallel to lines AB and CD.

The new line cuts angle y into two parts. The top part (●) is equal to x because the new line is parallel to AB. The bottom part (♦) is equal to z because the new line is parallel to CD.
So, altogether, y is equal to $x + z$.

So Emma says it's true

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Appendix C Y9 School Questionnaire

INSTITUTE OF EDUCATION
UNIVERSITY OF LONDON

School Questionnaire (Y9)

Name of person completing questionniare .....

School ..... LEA .....

Please complete this questionnaire at a convenient time and keep it with the other completed project materials.

Complete the details above and overleaf.

Sch

Longitudinal Proof Project

Funded by the Economic and Social Research Council

School data

Please tick the boxes which best describe your school

Selection		Single-sex/mixed		Area	
No academic selection	❑ 1	Girls-only	❑ 1	Urban	❑ 1
Some academic selection	❑ 2	Boys-only	❑ 2	Rural	❑ 2
Full academic selection	❑ 3	Mixed-sex	❑ 3	Suburban	❑ 3

Year 9 data

Approximate number of Y9 students in the school .....

How are the current Y9 classes organised? *Please tick one box*

Set ❑ 1 Banded ❑ 2 Mixed ability ❑ 3 Other ❑ 4

How many of the Y9 students taking the proof survey were entered for

the level 6 - 8 KS 3 test papers .....

the extension paper ? .....

Mathematics curriculum data

GCSE examination syllabus .....

Main textbook / scheme in Year 9 .....

Total duration (in minutes) of
Y9 mathematics lessons per week .....

Extra-curricular mathematics activities

Are any Y9 students involved (at school or elsewhere) in any mathematics activities that are not part of the normal school mathematics curriculum (eg maths club, master classes, UK Maths Challenge)?

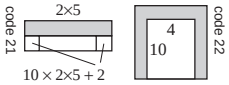
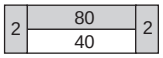
Yes ❑ 1 No ❑ 2

If YES, please describe:

.....

Appendix D Y9 Coding Scheme

Coding Sheet Year 9 Proof Survey (and scores) version 14 (retro) 10 December 2001

Qu	Description of response and key points	Score
A1a	*Generating data, spotting patterns, no structure* — Incorrect SCALAR (12): 4 → 12 (×10) → 40 → 120; Incorrect FUNCT'L (13): 4 → 12 (×3) → 40 → 120	
11	Answer = 120 + no reason or unclear	0
12	+ incorrect scalar strategy (10×12)	
13	+ uses incorrect functional str (3×40)	
14	+ incorrect scalar and functional	
	Some recognition of structure but incomplete or goes wrong or no reason or draws and counts	
21	Answer = blank/wrong: partial structure (eg doubles but does not add 4), or generates correct data (eg, 4,12 5,14, 6,16) but stops or goes wrong	1
	Answer = wrong: generates correct data (eg 5,14) but scales up (eg 5,14 × 8 gives 40,112)	
	Answer = wrong: sees correct structure initially but does not apply correctly, eg scales up inappropriately (eg 10 lots of 2×5, plus 1+1). 	
22	Answer = 30: sees white tiles as 10 rows of 4.	
23	Answer = 84 + no reason or no clear reason or possibly false reason	2
23T	Answer = 84 + erroneous use of (erroneous) table (eg sees white tiles in rows of 4, gets 4,12 and 8,14 and uses +2 → ×2 to get g=2w+4).	
24	Answer = 84 (or close to 84): eg, draws a **photo**-picture with 40 white tiles and counts	
	*Recognition and **use of** structure, specific*	
30	Answer = 84	3
	Schematic diagram (not photo) or description of **40** white tiles bridged by grey tiles. NOTHING MORE.  or **40** + **40** + 2 + 2, **40** × 2 + 4, 41 × 2 + 2	
30T	As code 30 but includes a table (or list) of data.	
	*Recognition and **use of** structure, <u>general</u>, SCALAR*	
41	Answer = 84: sees that there are 10 times as many white tiles as in the given diagram, so will need 10 times as many grey tiles as there are above the given white tiles, plus the 2 tiles at each end: $10 \times 8 + 4$. **No explicit naming of variables.** 4 → 8 + 4 (×10) → 40 → 80 + 4	3
41T	Answer = 84: makes a table, using the idea that for every <u>4 extra</u> white tiles that are <u>8 extra</u> grey tiles. May go all the way to 40, 84.	
	*Recognition and **use of** structure, <u>general</u>, FUNCTIONAL*	
42	Answer = 84	3
	Shows procedure for getting the number of grey from the number of white: eg, always two lots plus 4; double and add 4; 2 greys for each white; × 2 + 4; times 2 add 4. **No explicit naming of variables** (so eg does **not** use "white" to refer to *total* whites). Less emphasis on 40: concentrates on ops of × and +. might have similar drawing to code 30, or this: 	
42T	As code 42 but draws a table (or list) of data.	
	Towards Algebra: naming variables	
50	Answer = 84: as code 42 but also names one or both **variables** in **words** (and may expresses general relationship between variables): eg, The *number of grey* is 2 times the *number of white* plus 4, or Double the *amount of white tiles*, add 4, or Double the *white tiles*, add 4, or *white* × 2 add 4.	3
50T	As code 50, but uses a table (as in 30T, 41T, 42T).	
50L	As code 50, but includes use of **letters**, eg 2*w* + 4, or 2*w* add 4.	3
50LT	As code 50L, but uses a table.	
91	*No response* 92 *No time (or informative non response)* 93 *Miscellaneous*	0

white	grey
1	6
2	8
3	10
4	12

+ table = 30T

white	grey
4	12
8	20
12	28
16	36

41T table

white	grey
1	6
2	8
3	10
4	12

+ table = 42T

NOTE: Do not penalise *purely* arithmetical errors (eg 2×40 = 100) but add E to the code
Do not penalise *purely* counting errors (eg code 22, 24) but add E to the code
NEW for Y9: add T to all codes where *extra data* has been generated, not just where T is mentioned above

GENERAL NOTE (for all questions) Where students show some working but clearly give up so there's no answer, code as 93 (unless it is stated otherwise for a particular question)

Qu	Description of response and key points - BRIEF	Score
A1b	*Passive description or pattern spotting*	
11	Answer = 4n or 40n (or 4n=12g, 40n=84g, etc). Add W, Y.	
12	Answer = 3n. Add W, G, Y.	
13	Answer = 10n. Add W, G, Y.	0
	Partial structure, or correct structure inadequately expressed (no letter or letter as object)	
21	Answer = 2n. Add W, G, Y.	
22	Answer = 2n+2. Add W, G, Y.	
23	Answer = ×2+4 Add G (for 2g + 4, 2n + 4g)	1
	Correct structure, correctly expressed	
30	Answer = 2n + 4	
30W	Answer = 2w + 4	
30Y	Answer = 2white + 4	2
91	*No response*	
92	*No time or informative response*	
93	*Miscellaneous*	0

Add W for w,
G for g (**except for code 30**),
Y for White. Add P for Power (in particular for $n^2 + 4$). Add B for correct 'back to front' expression.
Treat all letters other than w and g the same as n. Treat "grey" the same as "g".

Qu	Description of response and key points - LONG	Score
A1b	*Passive description or pattern spotting*	
11	Answer = 4n or 40n (or 4n=12g, 40n=84g, etc).	
11W	Answer = 4w or 40w, etc.	
11Y	Answer = 4 white or 4 × white, etc.	
12	Answer = 3n.	
12W	Answer = 3w.	
12G	Answer = 3g.	
12Y	Answer = 3 white, or 3 × white, etc.	
13	Answer = 10n.	
13W	Answer = 10w.	
13G	Answer = 10g.	
13Y	Answer = 10 white, or 10 × white, etc.	
	Partial structure, or correct structure inadequately expressed (no letter or letter as object)	
21	Answer = 2n	
21W	Answer = 2w.	
21G	Answer = 2g.	
21Y	Answer = 2 white or 2 × white, etc.	
22	Answer = 2n+2.	
22W	Answer = 2w+2.	
22G	Answer = 2g+2 or 2n + 2g.	
22Y	Answer = 2 × white + 2.	
23	Answer = ×2+4.	
23G	Answer = 2g + 4, 2n + 4g.	
	Note: It is possible to have • code 22WG (2w+2g) and 22GY (2white+2g) • code 23WG (2w + 4g) and 23GY (2white + 4g)	
	Correct structure, correctly expressed	
30	Answer = 2n + 4	
30W	Answer = 2w + 4	
30Y	Answer = 2white + 4	

Note for Joyce: i. Answer may appear in explanation box, in which case, code it as normal.
ii. Take account of answers in **A2(a)i** but only as specified below.

Qu	Description of response and key points	Score
A2a ii 10	*Generating data; spotting number patterns* Answer = 120 + scales up from 4th pattern to 20th: 4th pattern is 24, so 20th pattern is 5×24. (Don't penalise arithmetic errors, eg 5×24=95, or if the number for the 4th pattern is wrong, but add E for error; but don't include other patterns, eg put "21×5−1" in code 93) [If there is no evidence of how 120 was obtained, give c93, or c91S if taken from part b).]	0
21	*Recognition of structure; ignoring Karen's method & adding or no reason or unclear* Answer = blank + gives up after starting to draw dot pattern, or starting on long additions (see code 22).	0.5
22	Answer = 440 + long additions 24 + 11 + 13 + ...etc. May be not quite complete so answer may be wrong, or may have made pure arithmetical error, in which case add E to code.	1
23	Answer = 440 (or 399 or 360 or 420) + **no explanation** or unclear method/explanation. However, code as 91S (S for Same) if answer clearly taken from part b).	1
31	*Recognition of structure: use of Karen's method: incorrect* Answer = 399: writes 20×20−1 Answer = 360: writes 19×19−1 (no longer using 'a' for 'alternative' as in Y8)	1.25
32	*Recognition of structure: use of Karen's method: correct* Answer = 440 Shows evidence of using Karen's method, by writing 21×21−1 or similar; may illustrate this with a picture, but shows nothing more explicit or general about the structure, ie just calculates. Add P for the Parallel correct structure 20×20+20+20 or 20×20+2×20 (but give code 91S for 20×22). **Note: Nothing more general or explicit written in A2(a)i**	1.5
40	*Recognition and use of structure, with structure made explicit* Answer = 440 As code 32, but with some explicit indication of structure, namely that 21 is 1 more than the pattern number, eg writes "20 + 1" rather than just "21" (or "4+1" rather than just "5"). Note: Might use narrative form. **This general method may appear in A2(a)i**	1.5
50	*Towards Algebra: naming variable* Answer = 440 As code 40, ie structure made explicit, and names variable, eg (number of pattern + 1) × (number of pattern + 1) − 1. [Add P for (number of pattern) × (number of pattern) + 2(number of pattern), or an equivalent Parallel expression.] Note: **This general method may appear in A2(a)i** (Do **not** include simple references to *base* and *height* in this code, such as Base×Height−1.)	1.5
50L	As code 50 but expresses the **general** structure in letters: eg, writes (n+1)×(n+1)−1 (but don't worry about the brackets). (Add P for $n^2 + 2n$, or an equivalent Parallel expression.) **Note: This general method may appear in A2(a)i** (Do **not** include simple references to *base* and *height* in this code, such as B×H−1.)	
91	*No response* [use 91S if answer taken straight from part b)]	0
92	*No time or informative response*	
93	*Miscellaneous*: Answers other than 120, 360, 399, 420, 441 (but not including pure arithmetic errors, and not including those obtained in code 22 by adding).	

NOTE: Do not penalise *purely* arithmetical errors (eg 21×21 = 402) but add E to the code;
Also add E to code for **forgetting to subtract 1**,
but only if there is clear evidence, in box aii, of the need to subtract 1 (if there is no evidence of "−1", give c93).

Note for Joyce: i. Answer may appear in explanation box, in which case, code it as normal.
ii. Take account of answers in **A2(b)i** but only as specified below.

Qu	Description of response and key points	Score
A2b ii	*Generating data; spotting number patterns*	
11	Answer = 120 + scales up from 4th pattern to 20th, by saying 4th pattern is 24, so 20th pattern is 5×24. (Don't penalise arithmetic errors, eg 5×24=95, or if the number for the 4th pattern is wrong, but add E for error; don't include other patterns, which should be coded 93, except 20×6 below.)	0
12	Answer = 120 + scales up from 4th pattern to 20th, by saying 4th pattern is 4×6, so 20th pattern is 20×6. (Add E for error, as for code 11.) [If there is no evidence of how 120 was obtained, give c93, or c91S if taken from part a).]	
	Recognition of structure; ignoring Josie's method & adding or no reason or unclear	
21	Answer = blank + gives up after starting to draw dot pattern, or starting on long additions (see code 22).	0.5
22	Answer = 440 + long additions 24 + 11 + 13 + ...etc. May be not quite complete so answer may be wrong, or may have made pure arithmetical error, in which case add E to code.	1
23	Answer = 440 (or 399 or 420 or 360) + **no explanation** or unclear method/explanation. However, code as 91S (S for Same) if answer clearly taken from part a).	1
	Recognition of structure: use of Josie's method: incorrect	
31	Answer = 399: writes 19×21 Answer = 420: writes 20×21 Answer = 360: writes 18×20 (no longer using 'a' for 'alternative' as in Y8)	1.25
	Recognition of structure: use of Josie's method: correct	
32	Answer = 440 Shows evidence of using Josie's method, by writing 20×22 or similar; may illustrate this with a picture, but shows nothing more explicit or general about the structure, ie just calculates. Add P for the Parallel correct structure 20×20+20+20 or 20×20+2×20 (but give code 91S for 21×21−1). **Note: Nothing more general or explicit written in A2(b)i**	1.5
	Recognition and use of structure, with structure made explicit	
40	Answer = 440 As code 32, but with some explicit indication of structure, namely that 22 is 2 more than the pattern number, eg writes "20 + 2" rather than just "22" (or "4+2" rather than just "6"). Note: Might use narrative form. **This general method may appear in A2(b)i**	1.5
	Towards Algebra: naming variable	
50	Answer = 440 As code 40, ie structure made explicit, and names variable, eg (number of pattern) × (number of pattern + 2). [Add P for (number of pattern) × (number of pattern) + 2(number of pattern), or an equivalent Parallel expression.] Note: **This general method may appear in A2(b)i** (Do **not** include simple references to *base* and *height* in this code, such as Base×Height.)	1.5
50L	As code 50 but expresses the **general** structure in letters: eg, writes n×(n+2) (but don't worry about the brackets). (Add P for $n^2 + 2n$, or an equivalent Parallel expression.) **Note: This general method may appear in A2(b)i** (Do **not** include simple references to *base* and *height* in this code, such as B×H.)	
91	*No response* [use 91S if answer taken straight from part a)]	0
92	*No time or informative response*	
93	*Miscellaneous*: Answers other than 120, 360, 399, 420, 441 (but not including pure arithmetic errors, and not including those obtained in code 22 by adding).	

NOTE: Do not penalise *purely* arithmetical errors (eg 20×22 = 462) but add E to the code.

Qu	Description of response and key points	Score
A2c i	*Passive description or pattern spotting*	
11	Answer = 6n.	
12	Answer = 5n.	0
	Expression of Karen's method using parameters	
20	Answer = L×W−1	0
	Correct algebraic expression of partially correct structuring of Karen's method	
31	Unambiguous version of $n^2 - 1$, for example $n^2 - 1$ or $n \times n - 1$ or of $(n-1)^2 - 1$, for example $(n-1)^2 - 1$ or $(n-1) \times (n-1) - 1$.	
31A	Ambiguous version of $n^2 - 1$, for example ??? or of $(n-1)^2 - 1$, for example $n-1 \times n-1 - 1$.	5/6 0.83
	Correct algebraic expression of correct structuring of Karen's method	
32	Unambiguous version of $(n+1)^2 - 1$, for example $(n+1)^2 - 1$ or $(n+1) \times (n+1) - 1$.	1
32A	Ambiguous version of $(n+1)^2 - 1$, for example $n+1 \times n+1 - 1$.	
	[Add P to code 32 or 32A for an ambiguous or unambiguous version of the Parallel $n^2 + 2n$, or equivalent (unless this is a simplification of another expression, in which case code the original expression).] (**Do not add E** to a code where the "− 1" at the end of the normal expression is omitted.)	
91	*No response* [use 91S if answer taken straight from part cii)]	
92	*No time or informative response*	
93	*Miscellaneous*	0

Where answer clearly derives from **part cii**, code it as 91S (S for Same).

Qu	Description of response and key points	Score
A2c ii	*Passive description or pattern spotting*	
11	Answer = 6n.	
12	Answer = 5n.	0
	Expression of Josie's method using parameters	
20	Answer = L×W	0
	Correct algebraic expression of partially correct structuring of Karen's method	
31	Unambiguous version of $(n-1) \times (n+1)$, for example $(n-1) \times (n+1)$ or $(n-1)(n+1)$, or of $n \times (n+1)$, for example $n \times (n+1)$ or $n(n+1)$, or of $(n-2) \times n$, for example $n \times (n-2)$ or $n(n-2)$.	
31A	Ambiguous version of $(n-1) \times (n+1)$, for example $n-1 \times n+1$, or of $n \times (n+1)$, for example $n \times n+1$, or of $(n-2) \times n$, for example $n \times n-2$.	5/6 0.83
	Correct algebraic expression of correct structuring of Karen's method	1
32	Unambiguous version of $n \times (n+2)$, for example $n \times (n+2)$ or $n(n+2)$.	
32A	Ambiguous version of $n \times (n+2)$, for example $n \times n+2$.	
	[Add P to code 32 or 32A for an ambiguous or unambiguous version of the Parallel $n^2 + 2n$, or equivalent (unless this is a simplification of another expression, in which case code the original expression).]	
91	*No response* [use 91S if answer taken straight from part ci)]	
92	*No time or informative response*	
93	*Miscellaneous*	0

Where answer clearly derives from **part ci**, code it as 91S (S for Same).

Qu	Description of response and key points	Score
A4a	*Replaces 5! by 5*	
10	Answer = No + 5 is not divisible by 3 (or because 5 is a prime number)	0
	Using definition and calculating	
30	Answer = Yes + Calculation: obtains answer **120** and knows 120 is divisible by 3.	3
30E	Answer = Yes/No + Calculation: pure arith error made in multiplication and/or in dividing by 3.	
	Using definition & understanding divisibility	
	Calculation not used in explanation *(if 120 appears it is not used in the explanation)*	
41	Answer = Yes + explanation (+ calculates 120 but does not use in explanation), eg "Times by 3 then divide by 3, so it is"; or writes 3×40 or $3 \times (5 \times 4 \times 2 \times 1)$.	3
42	Answer = Yes + explanation: eg, "$\times 3$ then $\div 3$, so it is"; or writes 3×40 or $3 \times (5 \times 4 \times 2 \times 1)$.	3
91	*No response* 92 *No time or informative response*	0
93	*Miscellaneous:* Answer = Yes or No + blank or anything not included in code 1 (eg *It* is not divisible by 3; Yes, 3 is in the middle of 5,4,3,2,1; Yes, $5! \div 3! = 20$; Yes, 5+4+3+2+1=15 which is divisible by 3; Yes, $5! = 5 \times 4 \times 4 \times 3 \times 2 \times 1 = 480$)	

A4b	*Imprecise or partial statement of definition or applied to a different number*	
11	eg, $50! = 50 \times 49 \times 48$ (ie stops short, with no indication that it should continue);	0
12	eg, $10! = 10 \times 9 \times 8 \times 7 \times \ldots \times 1$ (ie correct definition for a number other than 50).	
	Correct statement of definition	
31	Shows schematically that it 'goes down to 1' but implicit: eg, $50! = 50 \times 49 \times$ etc.	0.5
32	As code 31 and explicit, eg $50! = 50 \times 49 \times \ldots\ldots \times 4 \times 3 \times 2 \times 1$; or $50! = 50 \times 49 \times 48!$.	1
	For 31,32: Add 'N' for 'narrative' if the numbers are described rather than (or as well as) being listed, eg "multiply all the numbers up to 50" (code 32). For 32: Add 'Z' if zero not 1; add 'F' if written out in full.	
	Understands definition; expresses the structure in a general form	
41	Writes a general definition showing the **structure**, implicit to 1, eg $50 \times (50-1) \times (50-2) \times \ldots$	1
42	As 41 but explicit down to 1.	1
	For 41,42: Add 'Z' if zero not 1 Add 'N' if numbers expressed in narrative form, eg "Start at 50, keep subtracting 1 and multiply". Add 'L' if expressed in algebra, eg "$n! = n(n-1)(n-2) \ldots 3.2.1$, and $n = 50$".	
91	*No response* 92 *No time or informative response*	0
93	*Miscellaneous* Everything else in here, including number patterns (eg $50! = 50 \times 6$ because $4! = 4 \times 6$; or $50! = 10 \times 5!$; or $50! = 50 \times 40 \times 30 \times 20 \times 10$)	

A4c	*Tries to answer with incorrect or irrelevant reason, or "can't give a reason"*	
11	Answer = Yes or No + **some** coherent attempt to explain, but ignores definition. Uses 50, not 50!, eg "50 is not divisible by 19" or "50 is even, 19 is odd".	0
12	Uses number patterns, eg $50! = 10 \times 5!$ (but not $5 \times 5!$), or $100! = 100 \times 80 \times 60 \times 40 \times 20$.	
13	Answer = Yes or No or blank + states that **can't explain**, eg "I guessed" or "I can't do this mult". Answer = Yes or No + blank or 'no time' or 'no response' or a bit of working that is abandoned (ie code 13 = they recognise that they can't do it, and don't offer spurious explanation).	
	Uses definition correctly but reasons by induction	
20	Answer = Yes or No + attempt to explain by induction, with correct use of definition, eg "120 can be divided by 5,4,3,2,1 so I predict that 50! can be divided by 50,49,48,47,...19".	2
	Uses definition and understands divisibility	
40	Answer = Yes. **No calculation**; uses 'reversibility' argument, eg "Times by 19 so can divide by 19"; or "$19 \times$ (all other numbers) so 19 is factor".	3
40L	As code 40, but expresses argument in algebra, eg "$n!$ has as factors all the numbers less than n, and this includes 19 when $n = 50$".	
91	*No response:* blank + blank or blank + 'no response' 92 *No time:* blank + 'no time'.	0
93	*Miscellaneous:* Answer = Yes or No + unclear reason, eg "all answers are divisible by 19"; "19 does not go into *it*"; or spurious reason, eg "19 is not the middle number"; "19 is a prime number"; "the sum of the numbers is divisible by 19"; "50! is even, 19 is odd". [So 93 = spurious, but in this question gives up goes in 13.]	

Note to Joyce: Codes 91, 92, 93 not listed for this question, but use them in the usual way

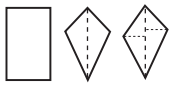
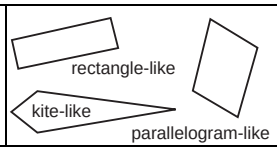
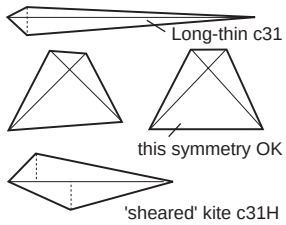
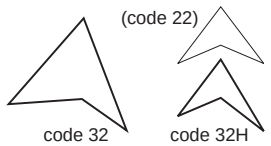
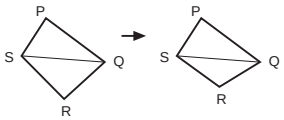
LA1	Yes	10	0		**LA1**	✓	sum is EVEN	30	2
(a)	Yes changed to No	31	1		**(b)**	✓	sum is ODD	93	0
	No	32	2			✓	can't be sure	10	0
						✓	more than one	93	0

L1c	*(Correct or incorrect) decision: no valid justification*	
11	Yes + nothing, or unclear* or vague (eg "If you test it it makes sense"; "tried examples"; repeats) + examples where condition does not hold (eg, 3,4) + some examples that confirm and some examples that deny (eg, 3,5 and 4,6).	0
13	No + nothing, or almost nothing *(but confused = 93, here and in what would otherwise be code 11)	
14	No + example where **condition** does **not** hold, ie sum not even (eg, 2+5=7 and 2×5=10, so product not odd; or "even × odd is not odd" (ie could be specific or general example)	
	Decision with incomplete or flawed justification	
21	No + mixture of examples: condition does not hold (as in code 14) **and** valid counter example (as in code 31,32) (specific or general)	2
22	Yes + examples that **confirm only**, ie only odd numbers (eg, 3+5=8 and/or 3×5=15) (spec or gen).	
23	No + valid counter example (eg, 2,4) but incomplete (ie might consider sum but **not** product).	
	(Basically) correct decision + correct justification	
31	No + implicit counter example (eg, "2+4=6 and 2×4=8" or just "2×4=8") but does **not** say why example is important; may include non counter-examples that fit condition of sum=even, (eg, 1+3=4, 1×3=3) [Note: ignore examples which do not satisfy the condition (eg, 2,5 as in code 2) if it is clear that these are not meant to be part of the answer] (specific only)	2.5
32	No + explicit counter example (eg, [2+4=6 and] 2×4=8, *and 8 is **even***) ie states *why* the example is a counter example [Note: ignore examples which do not satisfy the condition (eg, 2,5 as in code 2) if it is clear that these are not meant to be part of the answer]	3
	Correct decision + general justification/description in narrative form	
4	No + If (A) the sum is even, then (B) the numbers could be both even, then (C) the product would be even B = code 41, A+B = code 42, B+C = code 43, A+B+C = code 44.	3
+L	As codes 41 to 44 + algebraic description of the set of counter examples (eg E+E=E, E×E=E).	
	Correct decision + general justification plus explanation of why justification is true	
50	No + as code 4, and adds explanation of **why** two evens have an even sum and/or product.	3
50L	As code 50 but uses algebra (eg, $2x + 2y = 2[x+y]$, $2x \times 2y = 4xy$). (use of E or O not sufficient here)	

Add S for 'sometimes' to codes 22, 31, 32, 41, 42, 43, 44 when appropriate

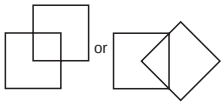
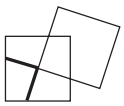
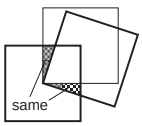
L1d	*(Correct or incorrect) decision: no valid justification*	
11	No + anything (including nothing)	0
13	Yes + nothing, or nothing sensible, or vague (eg "it always works"). + examples, some where condition does not hold, ie product not odd (eg, 2,4 or 3,4) (spec or gen) + repeat of statement (the product is odd, the sum is even) + false statement (specific or general)	
	Correct Decision + incomplete or limited justification, but not false	
21	Yes +confirmation by **one** empirical example	2
22	+confirmation by **several** empirical examples	
23	+confirmation by examples + recognition that this is not enough	
24	+crucial experiment i.e. random pair of odd numbers (eg, 19,23) [one or both numbers > 10].	
	Correct Decision + general justification of why numbers have to be odd + consequence:	
4	Yes + If (A) the product is odd, then (B) the numbers are both odd, then (C) the sum is even B = code 41, A+B = code 42, B+C = code 43, A+B+C = code 44.	3
+L	As codes 41 to 44 + algebraic description, eg "must be odd and O + O = E".	
	*Correct Decision + general justification plus explanation of **why** justification is true*	
50	Yes + As code 4 but adds explanation of **why** odd + odd is even (or why the numbers *have* to be odd)	3
50L	As code 50 but uses algebra (eg, $2x + 2y = 2[x+y]$, $2x \times 2y = 4xy$). (use of E or O not sufficient here)	

Parts c) and d):
try not to penalise pure arithmetic errors (but code as 93 if they lead to confusion);
also, code as 93 if question misunderstood [eg if they have not grasped the meaning of Sum and Product, or if they focus on the numbers that fit part b), or if they borrow from "You can't be sure ... until you know what the numbers are"];
code 12 abolished: For "same as Viv's/ Pam's" or "same as below/above", code the answer in the other box as if it had been written in the box you are currently coding, and add V (for L1c) or P (for L1d) to the code.

Qu	Description of response and key points	Score
G1 11	*Incorrect decision: confirming example or no explanation* Answer = Yes + anything (including nothing) May have picture of quadrilateral where a diagonal **does** cut area in half 	0
12	*Correct decision but no explanation* Answer = No + nothing or not clear or not sensible, or wrong (eg "could be parallelogram"). (Include answers that question the trustworthiness of Tim's sketch or sketches in general, but which say nothing about the quadrilateral itself, eg "Tim's is *not* cut in half", or "Can't tell unless we use a ruler and measure"[but this may be given a higher code if there is a diagram]).	1
21	*Correct but only implicit reasons: weak explanation (and no diagram)* Answer = No + ambiguous or weak description of a counter example (or family of counter examples), including reference (without a diagram) to 'trapezium' or 'irregular quad' or 'not symmetrical' (even though these don't *necessarily* give a valid counter example); however, code 'parallelogram' as c12 (as this *definitely* does not give a valid counter example). or + ambiguous or weak general explanation (global rather than analytic, ie concerned with the quadrilateral as a whole, rather than specifically with the endpoints of the diagonals), eg "quadrilaterals with different sides aren't symmetrical" "If two points are close together and the others far away, one region will be smaller" "If one side is much smaller, one region will be much smaller".	2
22	*Correct but only implicit reasons: weak diagram* Answer = No + ambiguous or weak diagram, ie the quadrilateral looks almost like a rectangle or kite (ie symmetrical along a diagonal) or parallelogram (but give 31H for 'sheared' kite) 	2
31	*Correct and explicit counter example* Answer = No + decisive diagram but decisiveness not 'absolute'; quadrilateral clearly not a rectangle, parallelogram, kite; possible reference to 'trapezium' or 'irregular' or 'not symmetrical'; accept small drawings (but give c12 or c21 if drawings too small to decipher); ignore diagonals (ie imagine *both* diagonals have been drawn and only give c31 if *neither* halves the area). Add H for same Height (eg, a sheared kite). 	2.5 3
32	Answer = No + clear description or drawing of concave quad (but code as 22 if an arrowhead or 'sheared' arrowhead, as in far diagram) [but no dynamic argument (see code 41)]. Code 32: Add D **or** N for Diagram **or** NoDiagram; Add H for same Height 	
41	*Correct analytic reason* Answer = No + use of dynamic argument, eg "If you move R you can change the area of QRS". Add D/N for Di/NoDi 	3
42	Answer = No + clear general explanation (not dynamic, which would be c41, but nonetheless analytic, ie concerned with the vertices, and in particular with their distances from the diagonals, rather than with the quadrilateral as a whole; but not simply a description of a counter example or family of counter examples), eg "R could be nearer than P to the diagonal SQ". Add D/N for Di/NoDi	3
91 92 93	*No response* *Informative no response* *Miscellaneous* (includes: illegible answers; diagram that does not satisfy conditions, eg draws pentagon; Yes *and* NO; *neither* Yes *nor* No; Q misunderstood, especially "at least")	0

For codes 32, 41, 42: letter D **or** N **must** be added

Qu	Description of response and key points	Score
G2a	*Incorrect*	
11	Answer = No + anything (including nothing) (including misunderstood or false argument)	0
	Says it must be, but no logical argument	
12	Answer = Yes + any invalid reason, eg no reason or 'Just must be' (tautology) or 'looks like it' or 'if you measure it' (perception) or 'both have two equal sides' (pseudo reason) or '*they* are the same' (ambiguous) or eg measures perimeter (= invalid). Add 'A' for 'always'	1
13	Answer = Yes + actual and valid measuring (eg draws square grid).	1
	Yes, by reference to specific example(s)	
20	Answer = Yes + refers to specific cases/orientations, eg triangles, or identical shapes with non-overlaps congruent, and/or makes statement like 'If 1/3 overlaps, then 2/3 does not overlap for each'; (*may* include reference to measuring, but no *actual* measuring ([see code 13]). Give this code even if other parts of the answer fit code 31 or 32. Add 'A' for 'always'.	2
	Yes by logical argument but incomplete (though does refer to overlap)	
31	Answer = Yes + says that the '**overlap is the same**' but not that the ovals have same area. Add 'A' for 'always'.	2.5
	Yes by logical argument which is complete, explicit, in narrative form	
32	Answer = Yes (Note: reference to 'ovals the same' **only**, would be code 12) + mentions '**overlap is the same**' and '**ovals have same area**' *somewhere* (ignore actual logic) (or equivalent explanation) [so the non-overlaps are the same] Add 'A' for 'always'	3
	Yes by logical argument, expressed in algebra	
32L	eg, A and B are the areas of ovals, X is the overlap; A = B, so A – X = B – X. Add 'A' for 'always'.	3
91	*No response* 92 *No time or informative response*	0
93	*Miscellaneous* (includes illegible answers; Yes *and* NO; *neither* Yes *nor* No)	

G2b	*Specific estimate, close but wrong*	
11	Answer = 1/3 or 1/5 (or decimal equivalent) + any or no explanation.	0
	Correct decision but no structural explanation	
12	Answer = $\frac{1}{4}$ + no explanation, or perception ("it looks like a quarter"), or spurious reason (the overlapping sides are halved and half times half is a quarter"). Add 'A' for 'always'.	1
13	Answer = $\frac{1}{4}$ + actual, valid measuring (eg draws grid and counts, or measures right angled triangle and calculates). Add E for answers that are close to but not exactly 1/4. Add 'A' for 'always'.	1
	Correct decision but only implicit reasons	
20	Answer = $\frac{1}{4}$ + sensible but only partial explanation (if obviously *not* sensible, then code 12). Could involve just one property ("corner is 90˚") but might involve several properties, and/or valid operations ("90˚ is a quarter of 360˚"; "You can divide the square into 4"); might include some reference to **turning** (but not as for code 31 or 32). Add 'A' for 'always'.	2
	Correct decision relating to case where obviously 1/4	
31	Answer = $\frac{1}{4}$ + refers to turning square D so that it is oriented as in one of these diagrams or draws one of the diagrams (eg turn it 'to the side' or 'to the bottom' or 'till it is parallel'). Add 'A' for 'always', Add D/N for Di/NoDi 	3
32	Answer = $\frac{1}{4}$ + claims that "the overlap fits 4 times", by referring to turning square D through successive 90˚ turns, or to partitioning the square into 4 equal parts, as in the diagram; or draws diagram. Add D/N for Di/NoDi 	3
	Explanation of 1/4 in general case	
40	Answer = $\frac{1}{4}$ + uses 'compensation' argument to explain **why** rotating from simple case (code 31) conserves the area of overlap ("on one side it is covering slightly more of the square and on the other the same amount less"). Add D/N for Di/NoDi 	3
91	*No response* 92 *Informative no response* 93 *Miscellaneous wrong resp* (not 1/3, 1/4, 1/5)	0

Add 'A' for 'always' throughout

G4a		Description of response and key points	Score
ABC	A	*Calculating angle u*	
	4	Result of '360 – *p*' (normally "40"), somewhere on the page, with some evidence of where it came from.	1
	3	"40", somewhere on the page, but with no evidence of where it came from.	1
	2	A value for *u* other than 40, due to a *factual* error concerning 'angle at a point', eg "angle at a point = 380˚", or to a *method* error (eg "???"), or to an unknown error (but if clearly due just to an arithmetic error, give the appropriate code and add E at the end).	0
	0	No discernible value for *u* anywhere on the page.	
	B	*Calculating v + w*	
	4	Result of '180 – *u*' (normally "140"), somewhere on the page, with some valid evidence of where it came from.	0
	3	Result of '180 – *u*' (normally "140"), somewhere on the page, but with no evidence of where it came from.	
	2	A value for *v + w* other than 140, due to a *factual* error concerning 'angle sum of a triangle', eg "angle sum = 360˚", or to a *method* error (eg "???") or to an unknown error (but if clearly due just to an arithmetic error, give appropriate code and add E at the end).	
	0	No discernible value for *v + w*.	
	C	*Calculating v*	
	4	Result of '÷2' (normally "70"), somewhere on the page, with some valid evidence of where it came from.	0
	3	Result of '÷2' (normally "70"), somewhere on the page, but with no evidence of where it came from.	
	2	A value for *v* other than 70, due to a *factual* error concerning 'base angles of an isosceles triangle', eg "??????????", or to a *method* error (eg "*v = u*") or to an unknown error (but if clearly due just to an arithmetic error, give the appropriate code and add E at the end).	
	0	No discernible value for *v*.	
91		*No response at all*	
92		*Informative no response*	
93		*Miscellaneous* C and CE scores:	2

Where a *viable* parallel method is used, add P for Parallel method; try to fit to ABC coding, else code as 93P.
Add C for Correct to code if final answer is "70" (unless clearly obtained by wrong method).
Add CE if final answer is correct apart from arithmetic error.

(bc)		Description of response and key points BRIEF	Score
ABC	A	*Reason for calculating u*	
	4	Correct mathematical reason, somewhere on the page, connected to calculation (ie immediately before or after calculation	1 1/3 (1.33)
	3	Correct mathematical reason, somewhere on the page, but not connected to calculation.	1
	2	Incorrect or unclear mathematical reason	
	1	Non-mathematical reason only.	
	0	No reason, anywhere on page.	
	B	*Reason for calculating v + w* Same coding as for A	same
	C	*Reason for calculating v* Same coding as for A	same
91		*No response at all* 92 *Informative no response* 93 *Miscellaneous*	0

~~Add V for an early v=w~~, ~~D for Description~~, F for Foresight, T for isosceles Triangle.

Caselaw:

a) Give c4 for "If 320 then 40" even though there is no actual calculation.
Give 002C for "70˚" only; give 93C for "70˚"+working that makes no sense.

bc) Allow "p + u = 360" but not "320 + 40 = 360" as a (correct) mathematical reason (ie c4 or c3).
Similarly, allow "u + v + w = 180" but not "140 + 40 = 180" as a mathematical reason.
Also, allow "If p = 320˚ then u must fill up to 360˚" but not "There is an angle of 320˚, but it should be 360˚...".
Allow "360 – 3200 = 40, u = 40" as a 'minimal' c1 reason, but *only* if it appears in bc) [this breaks the 'anywhere' rule].
Treat this as a c3 rather than a c4 for C:
"180˚-40˚=140". I did this because v and w are the same so if I took angle u away, all I needed was to divide by 2".

Give c424 rather than 404 for an answer like "360-320=40, 40/2=20=v": treat this as A=correct=4, B=wrong=2 rather than missing=0 (it implies v+w=40), C=correct=4 (v=w). On the other hand give c402 for an answer like "360-320=40, u=v so v=40": here "v+w" (B) has been completely bypassed, and has a wrong relationship for "v=" (namely v=u rather than v=w).

G4 (bc)		Description of response and key points LONG	Score
ABC	A	*Reason for calculating u*	
	4	Correct mathematical reason, somewhere on the page, <u>connected</u> to calculation (ie immediately before or after calculation). [eg "angle at a point is 360˚" or "angle in a circle is 360˚", but accept loose wording, eg "angle at a corner is 360˚", "area of a circle is 360˚", "$p + u = 360$".]	1 1/3 (1.33)
	3	Correct mathematical reason, somewhere on the page, but <u>not</u> connected to calculation (ie <u>not</u> immediately before or after calculation).	1
	2	Incorrect mathematical reason.	
	1	Non-mathematical reason only. (Add <u>F for Foresight</u>: "I did this to find u so I could find v" or "I started with p=320 as this is the only thing we know".)	
	0	No reason, anywhere on page.	
	B	*Reason for calculating v + w*	
	4	Correct mathematical reason, somewhere on the page, <u>connected</u> to calculation (ie immediately before or after calculation). [eg "angle sum of triangle is 180˚", but accept loose wording, eg "triangle is 180˚", "area of a triangle is 180˚", "$u + v + w = 180$".]	1 1/3 (1.33)
	3	Correct mathematical reason, somewhere on the page, but <u>not</u> connected to calculation (ie <u>not</u> immediately before or after calculation).	1
	2	Incorrect mathematical reason.	
	1	Non-mathematical reason only. (Add <u>F for Foresight</u>: "I did this to find $v + w$ so I could then find v".)	
	0	No reason, anywhere on page.	
	C	*Reason for calculating v*	
	4	Correct mathematical reason, somewhere on the page, <u>connected</u> to calculation (ie immediately before or after calculation). [eg "$v = w$". Add <u>T for isosceles Triangle</u>, where there is reference to "isosceles" or "two sides are the same"; allow loose wording, eg "AB and AC are parallel".]	1 1/3 (1.33)
	3	Correct mathematical reason, somewhere on the page, but <u>not</u> connected to calculation (ie <u>not</u> immediately before or after calculation).	1
	2	Incorrect mathematical reason.	
	1	Non-mathematical reason only. (Add <u>F for Foresight</u>, though this is unlikely on the last step...)	
	0	No reason, anywhere on page.	
91		*No response at all* 92 *Informative no response* 93 *Miscellaneous*	0

~~Add V for v=w, if this appears well before the "÷2" calculation, in part a) or b) or c).~~

~~Add D for a Descriptive reason, eg "I did 180 – 40 to find v and w" or "180 – 40 gives you v + w". This may occur as part of, in addition to, or instead of a mathematical reason.~~

<u>Add F</u> for Foresight for a reason that looks at least one step ahead, eg "I did 360 – 320 to find u so I could find $v + w$" (but not for "I did 360 – 320 to find u" which would just be D); include "I started with 320 because that is the only value we know". A Foresight reason is likely to be descriptive, but only use D as well as F if *other* reasons are descriptive.

Do not penalise arithmetic errors, but there is no need to add E in bc).

<u>Add T</u> for 'isosceles Triangle' reasons.

In c), code "calculation B, calculation C, maths reason B, maths reason C" as 'connected'.

Further notes, while doing actual coding:

bc): if a wrong method is used, but not explained 'mathematically' anywhere, get a code 1 here rather than a code 2 (though would get c2 in a) if one can infer method).

Treat "divide by 2 cos 2 angles" as non-mathe rather than wrong mathe reason.

B: Treat "180 - 40 = what's left of the triangle" as non-mathe rather than adequate-mathe, ie c1, not c3 or c4.

B: If students sidestep step B (v+w=), eg by claiming "v=u" then give 0 for B in parts a) and bc).

If, however, they imply a wrong relationship for "v+w", eg by claiming v = 40/2 or 320/2 (which suggest v+w=40 or v+w=320), then give 2 for B in a), even if "v+w" is not referred to explicitly. (done, 29.11.1 = need to check whether some older B=0 should be B=2.)

Note to Joyce: Codes 91, 92, 93 not listed for this question, but use them in the usual way

LG1	Yes	10	0		**LG1**	✓	both ACUTE	30	2
(a)	Yes changed to No	31	1		**(b)**	✓	not both ACUTE	93	0
	No	32	2			✓	can't be sure	10	0
						✓	more than one	93	0

LGc	*(Correct or incorrect) decision: no valid justification*	
11	Yes + nothing, or unclear* or vague (eg "If you test it it makes sense"; "tried examples"; repeats) + [examples where condition does not hold (eg, ??)] + some examples that confirm and some that deny (eg, 30˚,40˚,110˚ and 60˚,60˚,60˚).	0
13	No + nothing, or almost nothing (eg "If acute + acute, then acute"). *(but confused = 93, here and in what would otherwise be code 11)	
14	No + example where **condition** does **not** hold, ie does not start with two acute angles (eg "if 30˚ and 140˚, then third angle is 10˚ which is not obtuse") (could be specific or general example).	
	Decision with incomplete or flawed justification	
21	No + mixture of examples: condition does not hold (as in code 14) **and** valid counter example (as in code 31,32) (specific or general)	2
22	Yes + examples that **confirm only**, (eg, "30˚,30˚,120˚") (spec or gen) Add G for General (classic = "If acute + acute, then obtuse" = c22G, but also include part d) c4 type answer). Add B for Back to front deduction (eg "If 120˚, then 30˚+30˚" = c22B; "If obtuse, then acute + acute" = c22BG). Add V for Visual (eg a drawing of a confirming triangle, but with no specific values marked; or use of a visual argument, with or without diagram, eg "If you have a base with two acute angles, where they meet has to be obtuse"). Note: cannot have V *and* G.	
23	No + valid counter example (eg, "If 80˚+80˚") but incomplete (ie does not mention all the angles). Add N for "90˚-angled triangle" + nothing else (but give c32N for "90˚-angled triangle and two acute angles").	
	(Basically) correct decision + correct justification	
31	No + implicit counter example (eg, "80˚,80˚,20˚") but does **not** state that third angle is "acute" or "not obtuse"; may include non counter-examples that confirm (eg, "30˚,30˚,120˚"). [Note: ignore examples which do not satisfy the condition if it is clear that these are not meant to be part of the answer] (specific only).	2.5
32	No + explicit counter example (eg, "If 80˚+80˚ then 20˚, **and** 20˚ is acute/not obtuse") ie states *why* the example is a counter example. [Note: ignore examples which do not satisfy the condition if it is clear that these are not meant to be part of the answer.] (Add B for Back to front deduction) (can't see how this could appear here) Add M for Maximum (use of extreme values, 85˚-90˚ for acute angles, eg "89˚,89˚,2˚"). Add V for Visual (eg a drawing of a counter example, but don't add V if specific values are marked on the diagram). Add T for equilateral Triangle (eg "could have 60˚,60˚,60˚") Add N for "No + 90˚-angled triangle has two acute angles", but only c23N for "No + right angled triangle").	3
	Correct decision + general justification/description in narrative form	
4	No + If (A) two angles are both <90˚, and (B) their sum ≥ 90˚, then (C) the third angle ≤ 90˚ B = code 41, A+B = code 42, B+C = code 43, A+B+C = code 44. (Only give c44 if there is a clear awareness that the sum of the two acute angles *can be* < 90˚.) (Give c32N for "No + right angled triangle has two acute angles", but only c23N for "No + right angled triangle").	3
+L	As codes 41 to 44 + algebraic description of the set of counter examples (eg if X+Y≥90, then Z<90). Add B, V (although B unlikely, and V more likely under c2 and c3 than here; don't assume that a simple drawing with unspecified angles is 'general'). c4 answers involve a *range* of values, but this need be explicit in only one of the statements (A), (B), (C).	
	Correct decision + general justification plus explanation of why justification is true	
50 50L	Code 5 redundant here. DO NOT USE. Covered by c4R.	

~~Add S for 'sometimes' to codes 22, 31, 32, 41, 42, 43, 44 when appropriate~~

Caselaw

c23: "No, if 80˚+80˚, then acute"

c22BG: "Yes, can't have two obtuse"

A general response can be c22 in c) but has to be c13 or c4X in d) rather than c2.

c11 confirm and deny: treat "Yes, ((unless equi))" as 'Yes' ie as possible c22. (This is different from usual 'sometimes' response, which might well = "No" as here the answer is strongly weighted to "Yes".)

c23: code "80+80=160 so 3rd is acute" as c31, ie more than c23 but not fully explicit as actual value not given.

c14: eg, "No, what if the first is obtuse?"

"No, if both *just under* 90, only need another acute to make 180": read this as 'B' only (sum over 90) = c41.

LGd		
11	*(Correct or incorrect) decision: no valid justification* No + anything (including nothing) (unless c93, see below)	0
13	Yes + nothing, or nothing sensible, or vague (eg "it always works"). + examples, some where condition not hold, ie first angle not obtuse (eg, right triangle) (spec or gen) + repeat of statement ("If 1 obtuse, then 2 acute"), or repeat of statement back to front + false statement (specific or general) (unless c93, see below)	
	Correct Decision + incomplete or limited justification, but not false	
21	Yes + confirmation by **one** empirical example	2
22	+ confirmation by **several** empirical examples	
23	+ confirmation by examples + recognition that this is not enough	
24	+ crucial experiment i.e. 'random' obtuse angle (eg 147˚) [not ending in 0 or 5, not 90-95˚, not 113˚]. Add B Add V Add M. [Use M for Minimum values 90˚ - 95˚. However, if a *range* is implied (eg "smallest value is 91˚"), then it might fit c4 (but this does not apply in part c).] [If the acute angles are left indeterminate, still use c2 if a specific value is given to the obtuse angle (eg, "If the obtuse angle is 130˚, then the other two add up to only 50˚"]	
	Correct Decision + counter-counter example (proof by contradiction)	
31	Yes +argument that there can't be another obtuse angle (as angle sum would be > 180˚) (but "so other angles must be acute" left implicit).	2.5
32	Yes +argument that there can't be another obtuse angle (as angle sum would be > 180˚) (and "so other angles must be acute" made explicit). Add V (eg, add V for "Can't have two obtuse angles as lines would not meet to make 3rd angle").	3
	Correct Decision + general justification of why resulting angles have to be acute:	
4	Yes + If (A) one angle > 90˚, then (B) the sum of the other two ≤ 90, so (C) they are both acute B = code 41, A+BorC = code 42, B+C = code 43, A+B+C = code 44. (Give c13 or c93 for 'Yes+any mention of "right angled triangle"').	3
+L	As codes 41 to 44 + algebraic description, eg " eg if Z>90, then X+Y≤90". Add B (though B unlikely here) Add V (though V unlikely here) ~~Add M~~ (redundant here; only allow a minimal value in c4 if it is *stated* that it is a minimal value) c4 answers involve a *range* of values, but this need be explicit in only one of the statements (A), (B), (C).	
	*Correct Decision + general justification plus explanation of **why** justification is true*	
50 50L	Code 5 redundant here. DO NOT USE. Covered by c4R.	

Parts c) and d):
try not to penalise pure arithmetic errors (but code as 93 if they lead to confusion);
also, code as 93 if question misunderstood [eg if they have not grasped the meaning of Acute and Obtuse, or if they think the angle sum is 360˚, say, or if they focus on values that fit part b) (113˚), or if they borrow from "You can't be sure ... until you know what the angles are"];
code 12 abolished: For "same as Bob's/ Jack's" or "same as below/above", code the answer in the other box as if it had been written in the box you are currently coding, and add S (for L1c) or S (for L1d) to the code.
Sadly, do not have a code for a drawing of an obtuse angle + "so need two acutes", so dump in c13.
Comment under c2 (above): if *range* implied could be c4, but only if some *further* generality in answer, eg "…so both acute"; so "Smallest = 91, so 89 left, so could be 30 and 59" is just not enough for c4, goes in c21 instead.
General rule: if statement transformed (even though tautology) then could be more than c13. Examples:
"If one Ob then others must be Ac or it won't join up" = c13 (bit tough as "won't join" does add something… close to 32V, but stick with c13).
"If one is more than 90, then others must be less" is a transformation, and just about A+B = c42.
"Smallest ob is 91, leaving 89 so angles acute" is general enough for c4 rather than c2.