APPENDIX A   The Baseline Mathematics Test

Baseline Maths Test

final

Name ....   Class ....
       first name   surname

School ....   Boy or Girl ....

Date of Birth ....   Today's Date ....
             day   month   year   day   month   year

You have 25 minutes to complete this test. Do NOT use a calculator.

Each question gives you a CHOICE of answers, like this:

A. 12   B. 8   C. 2.5   D. 31

Circle the letter next to the correct answer, like this:

A. 12   (B.) 8   C. 2.5   D. 31

Read each question carefully and answer it as well as you can. If you are not sure about the answer to a question, circle the answer you think is best and go on to the next question.

---

1 The nine counters shown are placed in a jar and mixed. Mandy draws one counter from the jar. What is the probability that Mandy draws a counter with an even number?

A. $\frac{1}{9}$ B. $\frac{2}{9}$ C. $\frac{4}{9}$ D. $\frac{1}{2}$

---

2 Four children measured the width of a room by counting how many paces it took them to cross it. The chart shows their measurements. Who had the longest pace?

Name	Number of paces
Stephen	10
Erika	8
Anna	9
Charles	7

A. Stephen   B. Erika   C. Anna   D. Charles

---

3 If $3(x+5)=30$, then $x=$

A. 2   B. 5   C. 10   D. 95

---

4 In a discus-throwing competition the winning throw was 61.60 m. The second-place throw was 59.72 m. How much longer was the winning throw than the second-place throw?

A. 1.18 m   B. 1.88 m   C. 1.98 m   D. 2.18 m

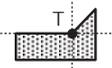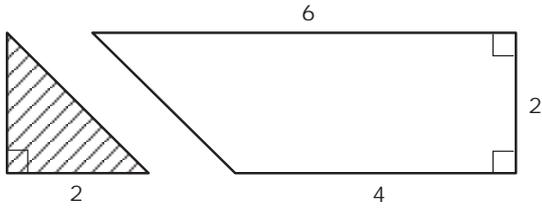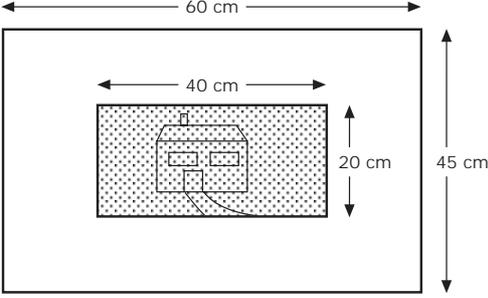**5** A drawer contains 28 pens: some black, some blue, some red, and some green.
If the probability of selecting a blue pen is $\frac{2}{7}$, how many blue pens are in the drawer?

A. 4   B. 6   C. 8   D. 10   E. 20

6 In this figure, lines AB and CD are parallel.
Two angles whose sizes must add up to 180˚ are

A. 1 and 3   B. 4 and 6   C. 2 and 5

D. 2 and 7   E. 1 and 8
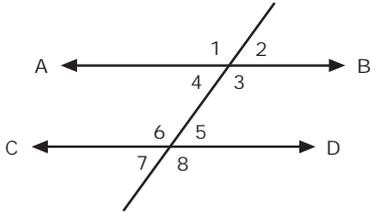
7 $P = LW$. If $P = 12$ and $L = 3$, then W is equal to

A. $\frac{3}{4}$ B. 3   C. 4   D. 12   E. 36

8 Last year there were 1172 students at Beaton High School. This year there are
15 per cent more students than last year. Approximately how many students are
at Beaton High School this year?

A. 1800   B. 1600   C. 1500   D. 1400   E. 1200

9 The graph shows the distance
travelled before coming to a
stop after the brakes are applied
for a typical car travelling at
different speeds.
A car travelling on a road
stopped 30 m after the brakes
were applied.
About how fast was the car
travelling?

A. 48 km per hour   B. 55 km per hour   C. 70 km per hour   D. 160 km per hour
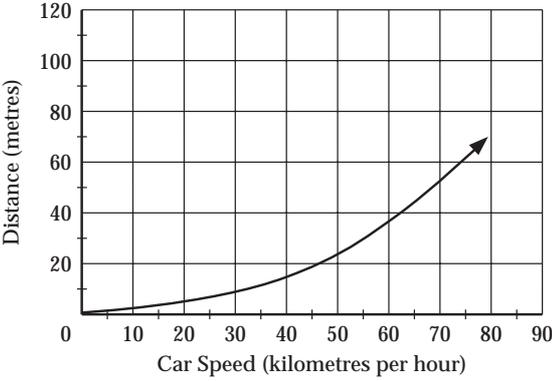
10 Which number is largest?

A. $\frac{4}{5}$ B. $\frac{3}{4}$ C. $\frac{5}{8}$ D. $\frac{7}{10}$

11	A half-turn about point T in the plane is applied to this figure. → Which of these shows the result of the half-turn? A. B. C. D. E.
12	Each of the six faces of a cube is painted either red or blue. When the cube is tossed, the probability of the cube landing with a red face up is $\frac{2}{3}$. How many faces are red? A. One B. Two C. Three D. Four E. Five
13	Which of these expressions is equivalent to y^3 ? A. $y+y+y$ B. $y \times y \times y$ C. $3y$ D. $y^2 + y$
14	How many triangles of the shape and size of the shaded triangle can the trapezium be divided into? A. Three B. Four C. Five D. Six
15	Which list shows the numbers from smallest to largest? A. 0.345, 0.19, 0.8, $\frac{1}{5}$ B. 0.19, $\frac{1}{5}$, 0.345, 0.8 C. 0.8, 0.19, $\frac{1}{5}$, 0.345 D. $\frac{1}{5}$, 0.8, 0.345, 0.19
16	In a quadrilateral, two of the angles each measure 110°, and the third angle measures 90°. What is the size of the remaining angle? A. 50° B. 90° C. 130° D. 140° E. None of these

|	
17	A stack of 200 identical sheets of paper is 2.5 cm thick. What is the thickness of one sheet of paper? A. 0.008 cm B. 0.0125 cm C. 0.05 cm D. 0.08 cm
18	Juan has 5 fewer hats than Maria, and Clare has 3 times as many hats as Juan. If Maria has n hats, which of these represents the number of hats that Clare has? A. $5 - 3n$ B. $3n$ C. $n - 5$ D. $3n - 5$ E. $3(n - 5)$
19	To mix a certain colour of paint, Alice combines 5 litres of red paint, 2 litres of blue paint, and 2 litres of yellow paint. What is the ratio of red paint to the total amount of paint? A. $\frac{5}{2}$ B. $\frac{9}{4}$ C. $\frac{5}{4}$ D. $\frac{5}{9}$
20	A rectangular picture is pasted to a sheet of white paper as shown. What is the area of the white paper not covered by the picture? A. 165 cm^2 B. 500 cm^2 C. 1900 cm^2 D. 2700 cm^2
21	If the price of a can of beans is raised from 60 pence to 75 pence, what is the percentage increase in the price? A. 15 % B. 20 % C. 25 % D. 30 %
22	Which one of the following is FALSE when a, b and c are different numbers? A. $(a + b) + c = a + (b + c)$ B. $ab = ba$ C. $a + b = b + a$ D. $(ab)c = a(bc)$ E. $a - b = b - a$
	If you have finished this test before the time is over then you may go back over your answers. *Thank you for taking the time to answer these questions carefully.*

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Year 8 Proof Survey

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Name ....   Class ....

first name   surname

School ....   Boy or Girl ....

Date of Birth ....   Today's Date ....

day   month   year   day   month   year

You have 50 minutes to answer these questions.

In two of the questions you will be asked to choose from a range of answers.

In all the other questions, you will be asked to produce your own answers. We are interested in your thinking as well as your answers, so please show all your rough working for these questions.

Put your rough working on the same page as your answer; use the answer box or any spare space on the page.

In most questions you will be asked for explanations. Make these as clear as you can, but don't make them longer than necessary.

Use a pen. You may cross things out, but do not rub out any of your work and do not use correction fluid.

Do not use a calculator.

You might find some of the questions quite difficult. Don't worry. If you get stuck on a question, leave it till later.

On the last page there is a questionnaire. Only fill this in if you have done all you can on the other questions and there is time left over.

A

Sch

Cla

Stu

Longitudinal Proof Project

A1 Lisa has some white square tiles and some grey square tiles.
They are all the same size.

She makes a row
of white tiles.

She surrounds the white
tiles by a single layer
of grey tiles.
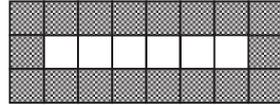
How many grey tiles does she need to surround a row of 60 white tiles? . . . .

Show how you obtained your answer.

G1   Darren sketches a circle.   He then draws a quadrilateral   He then draws the diagonals
     He calls the centre C.   PQRS, whose corners lie on   of the quadrilateral.
                                  the circle.
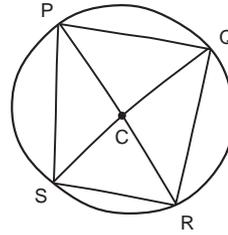
Darren says

"Whatever quadrilateral I draw with corners on a circle,
  the diagonals will always cross at the centre of the circle".

Is Darren right?   . . . .

Explain your answer.

2

A2 These are the first five patterns in a sequence of dotted triangle patterns:

1st
pattern

2nd
pattern

3rd
pattern

4th
pattern

5th
pattern
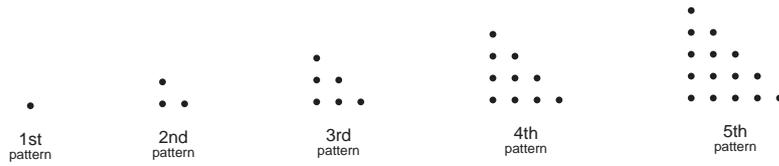
a) Rachel wants to *calculate* the number of dots in the 5th triangle pattern.

She draws the 5th pattern twice. She explains why:

"I can calculate the number of dots in this rectangle pattern.

I can then calculate the number of dots in the triangle pattern".
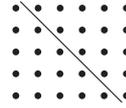
i. Show how Rachel calculates the number of dots in the rectangle pattern.

ii. Show how Rachel calculates the number of dots in the triangle pattern.

b) Rachel wants to find the number of dots in the 20th triangle pattern.
She imagines drawing it twice to produce a rectangle pattern.

Use the number of dots in the imagined rectangle pattern
to find the number of dots in the 20th triangle pattern.   . . . .
Show how you obtained your answer.

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3

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G3 In the diagram, A and B are two
fixed points on a straight line *m*.

Point P can move, but stays
connected to A and B
(the straight lines PA and
PB can stretch or shrink).

P

$z°$

$x°$ $y°$

A   B   *m*
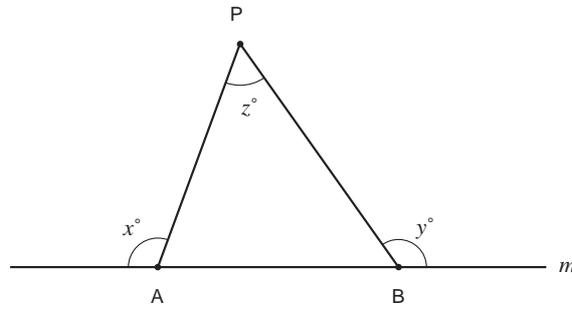
Avril, Bruno, Chandra and Don are discussing whether this statement is true:

$x° + y°$ is equal to $180° + z°$.

Avril's answer

I measured the angles in the diagram and
found that angle *x* is 110°, angle *y* is 125°
and angle *z* is 55°.

110° + 125° = 235°,

and 180° + 55° = 235°.

So Avril says it's true

Bruno's answer

I can move P so
that the triangle
is equilateral,
and its angles
are 60°.

60°

60°   60°

So *x* is 120° and *y* is 120°.

120° + 120° is the same as 180° + 60°.

So Bruno says it's true

Chandra's answer

I drew three parallel
lines.
The two angles marked
with a ● are the same
and the two marked
with a ○ are the same.

90°   90°

Angle *x* is 90°+ ● and angle y is 90°+ ○.

So *x* plus *y* is 180 + ● + ○, which is 180 + *z*.

So Chandra says it's true

Don's answer

I thought of a diagram where the angles *x*,
y and *z* are all 170°.

170°

170°   170°

So in my diagram *x* + *y* is not equal to 180 + *z*.

So Don says it's not true

---

a) Whose answer is closest to what you would do?   . . . .

b) Whose answer would get the best mark from your teacher?   . . . .

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c)   For each of the following, circle whether you agree, don't know, or disagree.

The statement is:
$x° + y°$ **is equal to 180°** $+ z°$.

	agree	don't know	disagree
Avril's answer ...			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3

	agree	don't know	disagree
Bruno's answer ...			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3

	agree	don't know	disagree
Chandra's answer ...			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3

	agree	don't know	disagree
Don's answer ...			
shows you that the statement is **not true**	1	2	3
shows you **why** the statement is not true	1	2	3

L1 Joe and Fred are thinking about the pair of numbers 3 and 11.

They notice that the SUM (3 + 11) is EVEN.

They notice that the PRODUCT (3 × 11) is ODD.

Joe says: If the SUM of two whole numbers is EVEN, their PRODUCT is ODD.

Fred says: If the PRODUCT of two whole numbers is ODD, their SUM is EVEN.

a) Are Joe's and Fred's statements saying the same thing? . . . .

b) The PRODUCT of two whole numbers is 1271.

Suppose Fred is right.

Which one of these must also be right? Tick (✓) one box.

❏ You can be sure that the SUM of the two numbers is EVEN.

❏ You can be sure that the SUM of the two numbers is ODD.

❏ You can't be sure whether the SUM is ODD or EVEN until
    you know what the two numbers are.

c) Is Joe's statement true? . . . .

Explain your answer.

d) Is Fred's statement true? . . . .

Explain your answer.

Please
leave
blank

A4 a) 4! means $4 \times 3 \times 2 \times 1$.
        5! means $5 \times 4 \times 3 \times 2 \times 1$.

        Is 5! exactly divisible by 3 ?   . . . .

        Explain your answer.

    b) What does 100! mean?

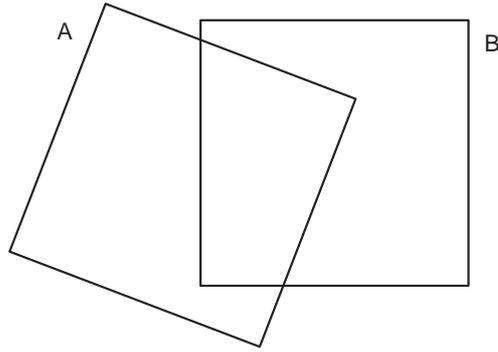   c) Is 100! exactly divisible by 31 ?   . . . .

        Explain your answer.

G2   The diagram shows two
      identical square tiles,
      A and B.

      The tiles overlap.

      a)   Do the two non-overlapping
            regions have the same area?

            . . . .

            Explain your answer.

      b)   One corner of tile A is moved
            to the centre of tile B, as shown.

            What fraction of tile B is
            overlapped by tile A ?

            . . . .

            Explain your answer.

91
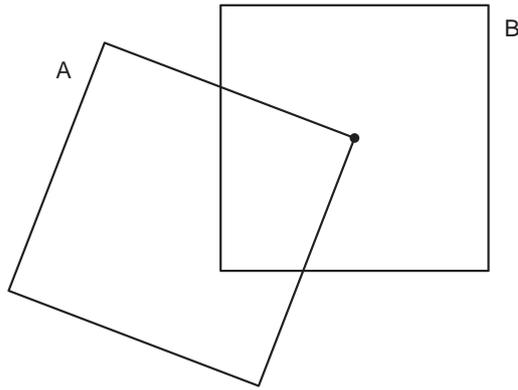
G4   In the diagram, ABC is a straight line and lines BD and BC are the same length.

D
v

u C

q
p
B

A
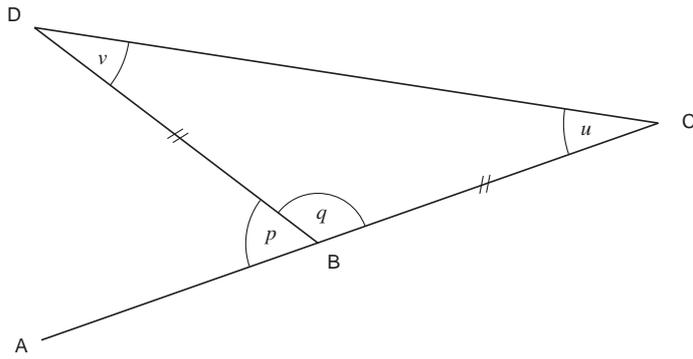
a)   Find the size of angle u when angle p is 70°.   . . . .

b)   Jenny wants to find the size of angle u when angle p is 76°.
      J, K and L are her calculations, but they are not in the right order.
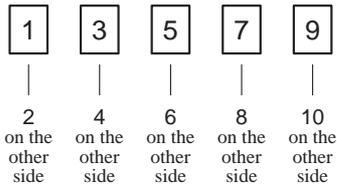
(J)   angle u + angle v = 180° − 104° = 76°

(K)   angle u = 76° ÷ 2 = 38°

(L)   angle q = 180° − 76° = 104°

      Write the letters J, K and L in the order in which Jenny did the calculations.   . . . .

c)   These are Jenny's reasons for her calculations, but they are not in the right order either.
      Match the reasons and calculations by writing the letters J, K, L in the blank circles:

◯   The base angles of an isosceles triangle are equal

◯   Angles on a straight line add up to 180°

◯   The angle sum of a triangle is 180°

9

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A3 Five cards have the odd numbers 1, 3, 5, 7 and 9
    printed on one side, and the even numbers
    2, 4, 6, 8 and 10 printed on the other side.

    The cards are dropped on the floor and spread out.

1	3	5	7	9
2 on the other side	4 on the other side	6 on the other side	8 on the other side	10 on the other side

Amina, Ben, Carol and Davor are discussing whether this statement is true:

When two of the visible numbers are even, the five visible numbers add up to 27.

Amina's answer

I tried this example:

1, 4, 6, 7, 9.

I then tried two more examples. Each had two even numbers and the total came to 27 each time. I could try other examples with two even numbers, they would come to 27 as well.

So Amina says it's true

Ben's answer

I tried all odd numbers first and got 25:

$1 + 3 + 5 + 7 + 9 = 25$.

If I change one odd number to an even number, the total will be 1 bigger.
So if I have two even numbers, the total will be 2 bigger.
So the total will be 27.

So Ben says it's true

Carol's answer

I wrote down these numbers:

1, 2, 3, 4, 9.

Two of the visible numbers are even but the total is 19. So you do not always get 27.

So Carol says it's not true

Davor's answer

I thought of these as the visible numbers:

1, 3, 6, 8, 9.

Two of them are even and when I add all the numbers I get 27.

So Davor says it's true

a)   Whose answer is closest to what you would do?   . . . .

b)   Whose answer would get the best mark from your teacher?   . . . .

10

A3   *Continued*

c)   For each of the following, circle whether you agree, don't know, or disagree.

The statement is:
When two of the visible numbers are even, the five visible numbers add up to 27.

	agree	don't know	disagree
Amina's answer ...			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3

	agree	don't know	disagree
Ben's answer ...			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3

	agree	don't know	disagree
Carol's answer ...			
shows you that the statement is **not true**	1	2	3
shows you **why** the statement is not true	1	2	3

	agree	don't know	disagree
Davor's answer ...			
shows you that the statement is **always true**	1	2	3
only shows you that the statement is true for **some** examples	1	2	3
shows you **why** the statement is true	1	2	3

WAIT! *Please go back to any questions you left out, then check all your answers.*
After that, if there is any time left over, please answer this questionnaire:

95

Z1   a)   What did you feel about taking part in this survey?

       b)   Which question did you like best, and why?

       c)   Which question did you like least, and why?

       d)   Please add any other comments, if you wish to, about the survey.

APPENDIX C   Teacher Questionnaire

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Teacher Questionnaire (Y8)

Name ....

School ....   LEA   ....

Name of your Y8 class with students involved in the survey   ....

Please complete this questionnaire while your students are
taking the proof survey.

Complete the details above and on pages 1 and 2, then work
through the proof questions that follow.

Sch [   ]

Cla [   ]

Tea [   ]

Longitudinal Proof Project

Please tick (✓) the appropriate boxes and complete the appropriate blanks

Female ☐ 1   Male ☐ 2

How many years teaching experience did you you have at the start of this school year?   ....

97

Your age: under 25 ☐ 1   25 - 29 ☐ 2   30 - 39 ☐ 3   40 - 49 ☐ 4   50 - 59 ☐ 5   60 or more ☐ 6

School responsibility: Head of maths ☐   Responsible for KS 3 ☐   Advanced skills teacher ☐

Mentor for beginning teachers ☐   Other (please specify)   ....

Teaching Qualification:

Please specify type of qualification and subjects studied

Type	Main subject (please specify)	Subsidiary subject (please specify)
Degree (if not BEd) ☐	....	....
BEd ☐	....	....
PGCE ☐	....	....
Cert Ed ☐	....	....
Other ☐	....	....

Higher Education (apart from above):

Please specify type of qualification and main subject studied

Type	Title (eg MEd)	Main subject	Year completed
Diploma ☐	....	....	....
Masters ☐	....	....	....
PhD ☐	....	....	....
Other ☐	....	....	....

1

Continuing Professional Development (CPD) or INSET in mathematics education

a. Involvement in Government-initiated INSET during 1999/2000:

    NNS: Have attended LEA's 2-day KS 3 National Numeracy Conference ☐

    NOF: Have attended New Opportunities Fund ICT training ☐

b. In this section do NOT include Government INSET for NNS or NOF, or any courses that you have mentioned in the Higher Education section, but DO include activities such as attending courses or conferences, writing text books, serving as an examiner, taking part in projects.

    Choose one school year from September 1995 to now.
    For that year, estimate the number of *sessions* you were involved in CDP or INSET in mathematics education (where a session is a morning, afternoon, twilight or evening):

	0	1 - 5	6 - 10	11 - 20	over 20
School based	☐1	☐2	☐3	☐4	☐5
LEA based	☐1	☐2	☐3	☐4	☐5
College based	☐1	☐2	☐3	☐4	☐5
Distance learning	☐1	☐2	☐3	☐4	☐5
Other	☐1	☐2	☐3	☐4	☐5

c. Current membership of a professional association:

    ATM ☐   MA ☐   IMA ☐   Other (please specify) ....

Involvement in extra-curricular mathematics activities with the students in your school during 1999/2000 (ie activites that are not part of the normal school mathematics curriculum):

Organise mathematics club ☐

Organise students for master classes or UK Maths Challenge ☐

Take students to mathematics events ☐

Other (please specify) ....

Software that you have used this school year with Y8 students:

Logo ☐   Dynamic geometry ☐   Spreadsheet ☐   Database ☐

Integrated learning system ☐   Other (please specify) ....

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A3   Five cards have the odd numbers 1, 3, 5, 7 and 9
     printed on one side, and the even numbers
     2, 4, 6, 8 and 10 printed on the other side.

| 1 | 3 | 5 | 7 | 9 |

2 on the other side   4 on the other side   6 on the other side   8 on the other side   10 on the other side
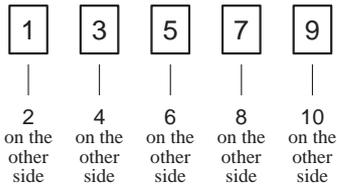
     The cards are dropped on the floor and spread out.

     Amina, Ben, Carol and Davor are discussing whether this statement is true:

     When two of the visible numbers are even, the five visible numbers add up to 27.

     a)   Please satisfy yourself that the statement above is true, then go on to part b).

     b)   Consider Amina, Ben, Carol and Davor's answers on the next page.

          i.   Give a mark (out of 10) for each answer.   A . . . .   B . . . .   C . . . .   D . . . .

          ii. Whose answer would your students say would get the best mark from you?   . . . .

          iii. Whose answer is closest to what you would do?   . . . .

     c)   Write a brief comment that might help these two students to move on:

          Amina . . . .

          . . . .

          . . . .

          . . . .

          Carol . . . .

          . . . .

          . . . .

          . . . .

Amina's answer

I tried this example:

1, 4, 6, 7, 9.

I then tried two more examples. Each had two even numbers and the total came to 27 each time. I could try other examples with two even numbers, they would come to 27 as well.

So Amina says it's true

Ben's answer

I tried all odd numbers first and got 25:

$1 + 3 + 5 + 7 + 9 = 25$.

If I change one odd number to an even number, the total will be 1 bigger.
So if I have two even numbers, the total will be 2 bigger.
So the total will be 27.

So Ben says it's true

Carol's answer

I wrote down these numbers:

1, 2, 3, 4, 9.

Two of the visible numbers are even but the total is 19. So you do not always get 27.

So Carol says it's not true

Davor's answer

I thought of these as the visible numbers:

1, 3, 6, 8, 9.

Two of them are even and when I add all the numbers I get 27.

So Davor says it's true

4

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G3 In the diagram, A and B are two fixed points on a straight line *m*.

Point P can move, but stays connected to A and B (the straight lines PA and PB can stretch or shrink).

Avril, Bruno, Chandra and Don are discussing whether this statement is true:

$x° + y°$ **is equal to** $180° + z°$.

a) Please satisfy yourself that the statement above is true, then go on to part b).

b) Consider Avril, Bruno, Chandra and Don's answers on the next page.

   i. Give a mark (out of 10) for each answer. A . . . .   B . . . .   C . . . .   D . . . .

   ii. Whose answer would your students say would get the best mark from you? . . . .

   iii. Whose answer is closest to what you would do? . . . .

c) Write a brief comment that might help these two students to move on:

   Avril . . . .

   . . . .

   . . . .

   . . . .

   Don . . . .

   . . . .

   . . . .

   . . . .

5

101

Avril's answer

I measured the angles in the diagram and found that angle x is 110˚, angle y is 125˚ and angle z is 55˚.

110˚ + 125˚ = 235˚,

and 180˚ + 55˚ = 235˚.

So Avril says it's true

Bruno's answer

I can move P so that the triangle is equilateral, and its angles are 60˚.
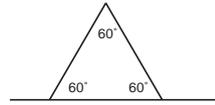
So x is 120˚ and y is 120˚.

120˚ + 120˚ is the same as 180˚ + 60˚.

So Bruno says it's true

Chandra's answer

I drew three parallel lines. The two angles marked with a ● are the same and the two marked with a ○ are the same.
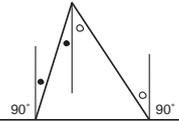
Angle x is 90˚+ ● and angle y is 90˚+ ○.

So x plus y is 180 + ● + ○, which is 180 + z.

So Chandra says it's true

Don's answer

I thought of a diagram where the angles x, y and z are all 170˚.

So in my diagram $x + y$ is not equal to 180 + z.

So Don says it's not true

6

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APPENDIX D   School Questionnaire

INSTITUTE OF EDUCATION
UNIVERSITY OF LONDON
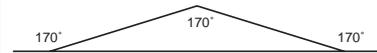
School Questionnaire (Y8)

Name of person completing questionniare ....

School ....   LEA   ....

Please complete this questionnaire at a convenient time and
keep it with the other completed project materials.

Complete the details above and overleaf.

Sch

Longitudinal Proof Project

School data

Please tick the boxes which best describe your school

Selection		Single-sex/mixed		Area	
No academic selection	☐¹	Girls-only	☐¹	Urban	☐¹
Some academic selection	☐²	Boys-only	☐²	Rural	☐²
Full academic selection	☐³	Mixed-sex	☐³	Suburban	☐³

Special status (eg, technology school, beacon school)   ....

Year 8 data

Approximate number of Y8 students in the school   ....

How are the current Y8 classes organised? *Please tick one box*

Set ☐¹   Banded ☐²   Mixed ability ☐³   Other ☐⁴

Give the approximate percentage of the Y8 students taking the
proof test who you predict will be entered for the level 6 - 8 KS 3 test   ....

Mathematics curriculum data

GCSE examination syllabus   ....

Main textbook / scheme in Year 8   ....

Total duration (in hours) of
Y8 mathematics lessons per week   ....

Extra-curricular mathematics activities

Are any Y8 students involved (at school or elsewhere) in any mathematics
activities that are not part of the normal school mathematics curriculum
(eg maths club, master classes, UK Maths Challenge)?

Yes ☐¹   No ☐²

If YES, please describe:

....

APPENDIX E Proof Survey Coding Scheme and Scoring Scheme

Coding Sheet Year 8 Proof Survey   29 September 2000 (+Pencilled bits agreed on 02.10.00)

Qu	Description of response and key points	Score
A1 11 12 13 14	*Generating data, spotting patterns, no structure* Incorrect SCALAR (12) Incorrect FUNCT'L (13) Answer = 180 + no reason or unclear + incorrect scalar strategy (10×18) + uses incorrect functional str (3×60) + incorrect scalar and functional Incorrect SCALAR: 6 18 ; $\times 10$; 60 180. Incorrect FUNCT'L: $6 \longrightarrow 18$; $\times 3$; $60 \longrightarrow 180$	0
21 22 23 23T 24	*Some recognition of structure but incomplete or goes wrong or no reason or draws and counts* Answer = blank/wrong: partial structure (eg doubles but does not add 6), or generates correct data (eg, 6,18 7,20, 8,22) but stops or goes wrong Answer = wrong: generates correct data (eg 10,26) but scales up (eg 10,26 $\times$ 6 gives 60,156) Answer = wrong: sees correct structure initially but does not apply correctly, eg scales up inappropriately (eg 2 lots of 10×8, plus 1+1). code 21: $10 \times 8 \times 2 + 2$; code 22 Answer = 36: sees white tiles as 10 rows of 6. Answer = 126 + no reason or no clear reason or possibly false reason Answer = 126 + erroneous use of (erroneous) table (eg sees white tiles in rows of 6, gets 6,18 and 16,20 and uses $+2 \to \times 2$ to get g=2w+6). Answer = 126 (or close to 126): eg, draws a **photo**-picture with 60 white tiles	1 2
30 30T	*Recognition and **use** of structure, specific* Answer = 126 Schematic diagram (not photo) or description of **60** white tiles surrounded by grey tiles. NOTHING MORE. As code 30 but includes a table (or list) of data. 3 60 3 ; 60 ; 60 or **60 + 60 + 3 + 3,** **60 $\times$ 2 + 6,** **62 + 62 + 2** white|grey: 1|8, 2|10, 3|12, 4|14 + table = 30T	3
41 41T	*Recognition and **use** of structure, general, SCALAR* Answer = 126: sees that there are 10 times as many white tiles as in the given diagram, so will need 10 times as many grey tiles as there are above and below the given white tiles, plus the 3 tiles at each end: $10 \times 12 + 6$. **No explicit naming of variables.** Answer = 126: makes a table, using the idea that for every 6 extra white tiles that are 12 extra grey tiles. May go all the way to 60, 126. 6 12 + 6 ; $\times 10$; 60 120 + 6 white | grey: 6|18, 12|30, 18|42, 24|56 41T table	3
42 42T	*Recognition and **use** of structure, general, FUNCTIONAL* Answer = 126 Shows procedure for getting the number of grey from the number of white: eg, always two lots plus 6; double and add 6; 2 greys for each white; $\times 2 + 6$; times 2 add 6. **No explicit naming of variables** (so eg does **not** use "white" to refer to *total* whites). Less emphasis on 60: concentrates on ops of $\times$ and +. As code 42 but draws a table (or list) of data. might have similar drawing to code 30, or this: white|grey: 1|8, 2|10, 3|12, 4|14 + table = 42T	3
50 50T 50L 50LT	*Towards Algebra: naming variables* Answer = 126: as code 42 but also names one or both **variables** in **words** (and may expresses general relationship between variables): eg, The *number of grey* is 2 times the *number of white* plus 6, or Double the *amount of white tiles*, add 6, or Double the *white tiles*, add 6, or *white* $\times$ 2 add 6. As code 50, but uses a table (as in 30T, 41T, 42T). As code 50, but includes use of **letters**, eg 2*w* + 6, or 2*w* add 6. As code 50L, but uses a table.	3 3
91	*No response* 92 *No time (or informative non response)* 93 *Miscellaneous*	0

NOTE: Do not penalise *purely* arithmetical errors (eg 2×60 = 100) but add E to the code

Do not penalise *purely* counting errors (eg code 22, 24) but add E to the code

GENERAL NOTE (for all questions) Where students show some working but clearly give up so there's no answer, code as 93 (unless it is stated otherwise for a particular question)

Qu	Description of response and key points	Score
A2b 10	*Generating data; spotting number patterns* <u>Answer</u> = 60 + scales up from 5th pattern to 20th: 5th triangle number is 15, so 20th triangle number is 15×4; or 5th *rectangle* pattern is 30, so 20th triangle number is $30 \times 4 \div 2$; or 5th triangle pattern is $6 \times 5 \div 2$, so 20th is $6 \times 20 \div 2$. (Don't penalise arithmetic errors, eg 15×4=45, but don't include other patterns, eg put "15×20" and "20th rectangle = 4×5 × 4×6 = 20×24" in code 93)	0

21
 	
 22
 	
 23
	Recognition of structure; ignoring Rachel's method & adding or no reason or unclear
	 <u>Answer</u> = blank + gives up after starting to draw triangular or rectangular dot pattern,
	      or starting on long additions (see code 22).
	 <u>Answer</u> = 210 + long additions 20 + 19 + 18 + ...etc. May be not quite complete so answer may
	      be wrong, or may have made pure arithmetical error, in which case <u>add E to code</u>.
	 <u>Answer</u> = 210 (or 200 or 190) + **no explanation** or unclear method/explanation.
		1
		
		 2
		
		 2

31 31a	*Recognition of structure: use of Rachel's method: incorrect* <u>Answer</u> = 200: writes $\frac{20 \times 20}{2}$ <u>Answer</u> = 190: writes $\frac{20 \times 19}{2}$ ('a' for 'alternative')	2.5
32	*Recognition of structure: use of Rachel's method: correct* <u>Answer</u> = 210 Shows evidence of using Rachel's method, by writing $\frac{20 \times 21}{2}$ or similar; may illustrate this with a picture, but shows nothing more explicit or general about the structure, ie just calculates. **Note: Nothing more general or explicit written in A2(a)**	3
40	*Recognition and use of structure, with structure made explicit* <u>Answer</u> = 210 As code 32, but with some explicit indication of structure, namely that one number <u>is 1 more than</u> the other, eg writes $20 \times (20 + 1) \div 2$, rather than just $20 \times 21 \div 2$. Note: Might use narrative form. **This general method may appear in A2(a)**	3
50 50L	*Towards Algebra: naming variable* <u>Answer</u> = 210 As code 40, ie structure made explicit, and names variable, eg (number of triangle) × (number of triangle + 1) ÷ 2. Note: **This general method may appear in A2(a)** (Do **not** include simple references to *base* and *height* in this code, such as $\frac{base \times height}{2}$) As code 50 but expresses the **general** structure in letters: writes $\frac{n \times (n+1)}{2}$. **Note: This general method may appear in A2(a)** (Do **not** include simple references to *base* and *height* in this code, such as $\frac{b \times h}{2}$.)	3
91 92 93	*No response* *No time or informative response* *Miscellaneous*: Answers other than 60, 190, 200, 210 (but not including pure arithmetic errors, and not including those obtained in code 22 by adding).	0

NOTE: Do not penalise *purely* arithmetical errors (eg 20×21 = 441) but add E to the code;
     also add E to code for **forgetting to divide by 2**

Qu	Description of response and key points	Score
A4a 10	*Replaces 5! by 5* <u>Answer</u> = No + 5 is not divisible by 3 (or because 5 is a prime number)	0
30 30E	*Using definition and calculating* <u>Answer</u> = Yes + Calculation: obtains answer **120** and knows 120 is divisible by 3. <u>Answer</u> = Yes/No + Calculation: pure arith error made in multiplication and/or in dividing by 3.	3
41 42	*Using definition & understanding divisibility* **Calculation not used in explanation** *(if 120 appears it is not used in the explanation)* <u>Answer</u> = Yes + explanation (+ calculates 120 but does not use in explanation), eg "Times by 3 then divide by 3, so it is"; or writes 3×40 or $3 \times (5 \times 4 \times 2 \times 1)$. <u>Answer</u> = Yes + explanation: eg, "$\times$ 3 then $\div$ 3, so it is"; or writes 3×40 or $3 \times (5 \times 4 \times 2 \times 1)$.	3 3
91 93	*No response 92 No time or informative response* *Miscellaneous:* <u>Answer</u> = Yes or No + blank or anything not included in code 1 (eg *It* is not divisible by 3; Yes, 3 is in the middle of 5,4,3,2,1; Yes, 5! ÷ 3! = 20; Yes, 5+4+3+2+1=15 which is divisible by 3; Yes, 5! = 5×4×4×3×2×1 = 480)	0

Qu	Description of response and key points	Score
A4b 11 12	*Imprecise or partial statement of definition or applied to a different number* eg, $100! = 100 \times 99 \times 98$ (ie stops short, with no indication that it should continue); eg, $10! = 10 \times 9 \times 8 \times 7 \times \ldots \times 1$ (ie correct definition for a number other than 100).	0
31 32	*Correct statement of definition* Shows schematically that it 'goes down to 1' but <u>implicit</u>: eg, $100! = 100 \times 99 \times$ etc. As code 31 and <u>explicit</u>, eg $100! = 100 \times 99 \times \ldots\ldots \times 4 \times 3 \times 2 \times 1$; or $100! = 100 \times 99 \times 97!$. For 31,32: <u>Add 'N' for 'narrative'</u> if the numbers are described rather than (or as well as) being listed, eg "multiply all the numbers up to 100" (code 22). For 32: <u>Add 'Z' if zero not 1</u>; add 'F' if written out in full.	0.5 1
41 42	*Understands definition; expresses the structure in a general form* Writes a general definition showing the **structure**, <u>implicit</u> to 1, eg 100×(100-1)×(100-2)×... As 41 but <u>explicit</u> down to 1. For 41,42: <u>Add 'Z'</u> if zero not 1 <u>Add 'N'</u> if numbers expressed in narrative form, eg "Start at 100, keep subtracting 1 and multiply". <u>Add 'L'</u> if expressed in algebra, eg "$n! = n(n-1)(n-2) \ldots 3.2.1$, and $n = 100$".	1 1
91 93	*No response 92 No time or informative response* *Miscellaneous* Everything else in here, including number patterns (eg 100! = 100×6 because 4! = 4×6; or 100! = 20×5!; or 100! = 100×80×60×40×20)	0

Qu	Description of response and key points	Score
A4c 11 12 13	*Tries to answer with incorrect or irrelevant reason, or "can't give a reason"* <u>Answer</u> = Yes or No + **some** coherent attempt to explain, but ignores definition. Uses 100, not 100!, eg "100 is not divisible by 31" or "100 is even, 31 is odd". Uses number patterns, eg $100! = 20 \times 5!$ (but not $5 \times 5!$), or $100! = 100 \times 80 \times 60 \times 40 \times 20$. <u>Answer</u> = Yes or No or blank + states that **can't explain**, eg "I guessed" or "I can't do this mult". <u>Answer</u> = Yes or No + blank or 'no time' or 'no response' or a bit of working that is abandoned (ie code 13 = they recognise that they can't do it, and don't offer spurious explanation).	0
20	*Uses definition correctly but reasons by induction* <u>Answer</u> = Yes or No + attempt to explain by induction, with correct use of definition, eg "120 can be divided by 5,4,3,2,1 so I predict that 100! can be divided by 100,99,98,97,...31".	2
40 40L	*Uses definition and understands divisibility* <u>Answer</u> = Yes. **No calculation**; uses 'reversibility' argument, eg "Times by 31 so can divide by 31"; or "31 × (all other numbers) so 31 is factor". As code 40, but expresses argument in algebra, eg "$n!$ has as factors all the numbers less than n, and this includes 31 when $n = 100$".	3
91 93	*No response:* blank + blank or blank + 'no response' 92 *No time:* blank + 'no time'. *Miscellaneous:* <u>Answer</u> = Yes or No + unclear reason, eg "all answers are divisible by 31"; "31 does not go into *it*"; or <u>spurious</u> reason, eg "31 is not the middle number"; "31 is a prime number"; "the sum of the numbers is divisible by 31"; "100! is even, 31 is odd". [So 93 = <u>spurious</u>, but in this question <u>gives up</u> goes in 13.]	0

Note to Joyce: Codes 91, 92, 93 not listed for this question, but use them in the usual way

L1 **(a)**	Yes	10	0		**L1** **(b)**	✓ ✓ ✓ ✓	sum is EVEN sum is ODD can't be sure more than one	30 93 10 93	2 0 0 0
	Yes changed to No	31	1						
	No	32	2						

L1c	*(Correct or incorrect) decision: no valid justification*	
11	<u>Yes</u> + nothing, or unclear* or vague (eg "If you test it it makes sense"; "tried examples"; repeats) + examples where condition does not hold (eg, 3,4) + some examples that confirm and some examples that deny (eg, 3,5 and 4,6).	0
13	<u>No</u> + nothing, or almost nothing *(but <u>confused = 93</u>, here and in what would otherwise be code 11)	
14	<u>No</u> + example where **condition** does **not** hold, ie sum not even (eg, 2+5=7 and 2×5=10, so product not odd; or "even × odd is not odd" (ie could be specific or general example)	
	Decision with incomplete or flawed justification	
21	<u>No</u> + mixture of examples: <u>condition does not hold</u> (as in code 14) **and** <u>valid counter example</u> (as in code 31,32) (specific or general)	2
22	<u>Yes</u> + examples that **confirm only**, ie only odd numbers (eg, 3+5=8 and/or 3×5=15) (spec or gen).	
23	<u>No</u> + valid counter example (eg, 2,4) but incomplete (ie might consider sum but **not** product).	
	(Basically) correct decision + correct justification	
31	<u>No</u> + implicit counter example (eg, "2+4=6 and 2×4=8" or just "2×4=8") but does **not** say why example is important; may include non counter-examples that fit condition of sum=even, (eg, 1+3=4, 1×3=3) [Note: ignore examples which do not satisfy the condition (eg, 2,5 as in code 2) if it is clear that these are not meant to be part of the answer] (specific only)	2.5
32	<u>No</u> + explicit counter example (eg, [2+4=6 and] 2×4=8, *and 8 is **even***) ie states *why* the example is a counter example [Note: ignore examples which do not satisfy the condition (eg, 2,5 as in code 2) if it is clear that these are not meant to be part of the answer]	3
	Correct decision + general justification/description in narrative form	
4	<u>No</u> + If <u>(A) the sum is even</u>, then <u>(B) the numbers could be both even</u>, then <u>(C) the product would be even</u> B = code 41, A+B = code 42, B+C = code 43, A+B+C = code 44.	3
+L	As codes 41 to 44 + algebraic description of the set of counter examples (eg E+E=E, E×E=E).	
	Correct decision + general justification plus explanation of why justification is true	
50	<u>No</u> + as code 4, and adds explanation of **why** two evens have an even sum and/or product.	3
50L	As code 50 but uses algebra (eg, 2x + 2y = 2[x+y], 2x×2y = 4xy). (use of E or O not sufficient here)	

<u>**Add S for 'sometimes'**</u> to codes 22, 31, 32, 41, 42, 43, 44 when appropriate

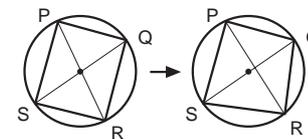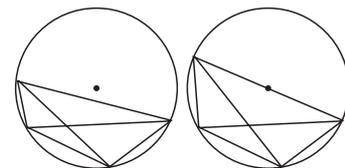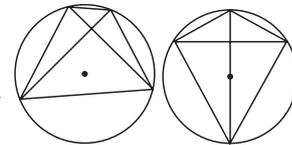| **L1d**	*(Correct or incorrect) decision: no valid justification*	
11	<u>No</u> + anything (including nothing)	0
13	<u>Yes</u> + nothing, or nothing sensible, or vague (eg "it always works"). + examples, some where condition does not hold, ie product not odd (eg, 2,4 or 3,4) (spec or gen) + repeat of statement (the product is odd, the sum is even) + false statement (specific or general)	
	Correct Decision + incomplete or limited justification, but not false	
21	<u>Yes</u> +confirmation by **one** empirical example	2
22	+confirmation by **several** empirical examples	
23	+confirmation by examples + recognition that this is not enough	
24	+crucial experiment i.e. random pair of odd numbers (eg, 19,23) [one or both numbers > 10].	
	Correct Decision + general justification of why numbers have to be odd + consequence:	
4	<u>Yes</u> + If <u>(A) the product is odd</u>, then <u>(B) the numbers are both odd</u>, then <u>(C) the sum is even</u> B = code 41, A+B = code 42, B+C = code 43, A+B+C = code 44.	3
+L	As codes 41 to 44 + algebraic description, eg "must be odd and O + O = E".	
	*Correct Decision + general justification plus explanation of **why** justification is tue*	
50	<u>Yes</u> + As code 4 but adds explanation of **why** odd + odd is even (or why the numbers *have* to be odd)	3
50L	As code 50 but uses algebra (eg, 2x + 2y = 2[x+y], 2x×2y = 4xy). (use of E or O not sufficient here)	

Parts c) and d):

try not to penalise pure arithmetic errors (but code as 93 if they lead to confusion);

also, code as 93 if question misunderstood [eg if they have not grasped the meaning of Sum and Product, or if they focus on the numbers that fit part b), or if they borrow from "You can't be sure ... until you know what the numbers are"];

<u>code 12 abolished</u>: For "same as Fred's/Joe's" or "same as below/above", code the answer in the <u>other</u> box as if it had been written in the box you are currently coding, and <u>add F (for L1c) or J (for L1d) to the code</u>.

Qu	Description of response and key points	Score
G1 11	*Incorrect decision: confirming example or no explanation* Answer = Yes + anything (including nothing) May have picture of quadrilateral where diagonals **do** cross at the centre	0
12	*Correct decision but no explanation* Answer = No + nothing or not clear or not sensible. (Include answers that question the trustworthiness of the sketch but which say nothing about the quadrilateral itself, eg "Can't tell unless we use ruler and compass"[but this may be given a higher code if there is a diagram]).	1

21	*Correct but only implicit reasons: weak explanation*
	 Answer = No
	 + ambiguous or weak description of a counter example (or family of counter examples), including reference (without a diagram) to 'trapezium' or 'kite'.
	   or
	 + ambiguous or weak general explanation (global rather than analytic, ie concerned with the quadrilateral as a whole, rather than specifically with the endpoints of the diagonals),
	   eg "quadrilaterals with different sides aren't symmetrical"
	     "quadrilaterals don't all have right angles"
	     "If the sides are much smaller on one side, the diagonals won't cross in the middle".
		2

22	*Correct but only implicit reasons: weak diagram* Answer = No + ambiguous or weak diagram (ie the quadrilateral looks almost like a rectangle and the centre is near but not at the intersection of the diagonals, and there are no constraints on the quadrilateral, as opposed to 31).	2
31	*Correct and explicit counter example* Answer = No + decisive diagram but decisiveness not 'absolute', (ie does not show that it would *never* be possible for the diagonals to meet at the centre with such a picture); quadrilateral clearly not a rectangle; possible reference to 'trapezium' or 'kite'; accept drawing with one diagonal going through centre, but only if not rectangle-like.	2.5
32	Answer = No + clear description of counter example or absolutely decisive diagram(s) [but no dynamic argument (see code 41)]. Code 32: Add D **or** N for Diagram **or** NoDiagram	3
41	*Correct analytic reason* Answer = No + use of dynamic argument, eg "One of the points (R) may be slightly offset, so the diagonal (PR) no longer goes through the centre". Add D/N for Di/NoDi	3
42	Answer = No + clear general explanation (not dynamic=41 but nonetheless analytic, ie concerned with the endpoints of the diagonals rather than with the quadrilateral as a whole; but not simply a description of a counter example or family of counter examples), eg "The corners could be anywhere and the diagonals will not necessarily go through centre". Add D/N for Di/NoDi	3
91 92 93	*No response* *Informative no response* *Miscellaneous* (includes: illegible answers; diagram that does not satisfy conditions, eg draws arrowhead, where one vertex not on circle; Yes *and* NO; *neither* Yes *nor* No)	0

For codes 32, 41, 42: letter D **or** N **must** be added

Qu	Description of response and key points	Score
G2a 11	*Incorrect* Answer = No + anything (including nothing) (including misunderstood or false argument)	0
12 13	*Says it must be, but no logical argument* Answer = Yes + any invalid reason, eg no reason or 'Just must be' (tautology) or 'looks like it' or 'if you measure it' (perception) or 'both have two equal sides' (pseudo reason) or '*they* are the same' (ambiguous) or eg measures perimeter (= invalid). Add 'A' for 'always' Answer = Yes + actual and valid measuring (eg draws square grid or right angled triangles).	1 1
20	*Yes, by reference to specific example(s)* Answer = Yes + refers to specific cases/orientations, eg triangles, or parallel squares (so the non-overlaps are congruent), and/or makes statement like 'If 1/3 overlaps, then 2/3 does not overlap for each'; (*may* include reference to measuring, but no *actual* measuring ([see code 13]). Give this code even if other parts of the answer fit code 31 or 32. Add 'A' for 'always'.	2
31	*Yes by logical argument but incomplete (though does refer to overlap)* Answer =Yes + says that the **overlap is the same** but not that the squares are the same. Add 'A' for 'always'.	2.5
32	*Yes by logical argument which is complete, explicit, in narrative form* Answer = Yes (Note: reference to 'squares the same' **only**, would be code 12) + mentions **'overlap is the same'** and **'squares are the same'** somewhere in explanation (or equivalent explanation) [so the non-overlaps are the same] Add 'A' for 'always'	3
32L	*Yes by logical argument, expressed in algebra* eg, A and B are the areas of squares, X is the overlap; A= B, so A − X = B − X. Add 'A' for 'always'.	3
91 93	*No response* 92 *No time or informative response* *Miscellaneous* (includes illegible answers; Yes *and* NO; *neither* Yes *nor* No)	0

Qu	Description of response and key points	Score
G2b 11	*Specific estimate, close but wrong* Answer = 1/3 or 1/5 (or decimal equivalent) + any or no explanation.	0
12 13	*Correct decision but no structural explanation* Answer = $\frac{1}{4}$ + no explanation, or perception ("it looks like a quarter"), or spurious reason (the overlapping sides are halved and half times half is a quarter"). Add 'A' for 'always'. Answer = $\frac{1}{4}$ + actual, valid measuring (eg draws grid and counts, or measures right angled triangle and calculates). Add E for answers that are close to but not exactly 1/4. Add 'A' for 'always'.	1 1
20	*Correct decision but only implicit reasons* Answer = $\frac{1}{4}$ + sensible but only partial explanation (if obviously *not* sensible, then code 12). Could involve just one property ("corner is 90˚") but might involve several properties, and/or valid operations ("90˚ is a quarter of 360˚"; "You can divide the square into 4"); might include some reference to **turning** (but not as for code 20 or 32). Add 'A' for 'always'.	2
31	*Correct decision relating to case where obviously 1/4* Answer = $\frac{1}{4}$ + refers to turning square A so that it is oriented as in one of these diagrams or draws one of the diagrams (eg turn it 'to the side' or 'to the bottom' or 'till it is parallel'). Add 'A' for 'always', Add D/N for Di/NoDi	3
32	Answer = $\frac{1}{4}$ + claims that "the overlap fits 4 times", by referring to turning square A through successive 90˚ turns, or to partitioning the square into 4 equal parts, as in the diagram; or draws diagram. Add D/N for Di/NoDi	3
40	*Explanation of 1/4 in general case* Answer = $\frac{1}{4}$ + uses 'compensation' argument to explain **why** rotating from simple case (code 31) conserves the area of overlap ("on one side it is covering slightly more of the square and on the other the same amount less"). Add D/N for Di/NoDi	3
91	*No response* 92 *Informative no response* 93 *Miscellaneous wrong resp* (not 1/3, 1/4, 1/5)	0

Add 'A' for 'always' throughout

Qu	Description of response and key points	Score
G4a 20	*Partially correct calculation* <u>Answer</u> = 55˚ or 110˚ or <u>Answer</u> = anything + evidence of calculating 110˚ somewhere on the page (unless there is sufficient evidence to be sure the wrong answer is only due to an arithmetic error [see code 30])	1
30	*Correct calculation* <u>Answer</u> = 35˚ Do not penalise arithmetic errors if all the steps are shown and it is clear that the wrong answer is due only to an arithmetic error, eg, "180 -70 = 130; 130/2 = 65; answer = 65°" or ".... 70/2 = 45". In such cases, add 'E' for 'error'.	2
91 92 93	*No response* *Informative no response* *Miscellaneous wrong responses (include 38 and 35.5, but not 55 or 110 or answers due only to arithmetic errors)*	0

(b)	*Correct*	
30	L J K	2
91 92 93	*No response* *Informative no response* *Miscellaneous wrong responses*	0

(c)	*Correct*	
30	K L J	2
91 92 93	*No response* *Informative no response* *Miscellaneous wrong responses*	0