Appendix A  Y10 Proof Survey

You have 55 minutes to answer these questions.

In two of the questions you will be asked to choose from a range of answers.

In all the other questions, you will be asked to produce your own answers. We are interested in your thinking as well as your answers, so please show all your rough working for these questions.

Put your rough working on the same page as your answer; use the answer box or any spare space on the page.

In most questions you will be asked for explanations. Make these as clear as you can, but don't make them longer than necessary.

Use a pen. You may cross things out, but do not rub out any of your work and do not use correction fluid.

Do not use a calculator.

The questions are not ordered by difficulty. If you get stuck on a question, don't worry - leave it till later.

On the last page there is a questionnaire. Only fill this in if you have done all you can on the other questions and there is time left over.

Longitudinal Proof Project

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1 Lisa has some white square tiles and some grey square tiles. They are all the same size.

She makes a row of white tiles.

She surrounds the white tiles by a single layer of grey tiles.

a) How many grey tiles does she need to surround a row of 60 white tiles? 

Show how you obtained your answer.

b) Write an expression for the number of grey tiles needed to surround a row of $n$ white tiles.
Vincent sketches a circle. He calls the centre C. He then draws a quadrilateral PQRS, whose corners lie on the circle. He then draws the diagonals of the quadrilateral.

Vincent says

“Whatever quadrilateral I draw with corners on a circle, the diagonals will always cross at the centre of the circle”.

Is Vincent right? 

Explain your answer.
Joe and Fred are thinking about the pair of numbers 5 and 9.

They notice that the SUM (5 + 9) is EVEN.

They notice that the PRODUCT (5 × 9) is ODD.

Joe says: If the SUM of two whole numbers is EVEN, their PRODUCT is ODD.
Fred says: If the PRODUCT of two whole numbers is ODD, their SUM is EVEN.

a) Are Joe’s and Fred’s statements saying the same thing? . . . . . . .

b) The PRODUCT of two whole numbers is 1247.

Suppose Fred is right.

Which one of these must also be right? Tick (✓) one box.

☐ You can be sure that the SUM of the two numbers is EVEN.
☐ You can be sure that the SUM of the two numbers is ODD.
☐ You can’t be sure whether the SUM is ODD or EVEN until you know what the two numbers are.

c) Is Joe’s statement true? . . . . . . .

Explain your answer.

☐

d) Is Fred’s statement true? . . . . . . .

Explain your answer.

☐
Aysha, Brian, Coby, Deon, Eric and Fiona were trying to prove whether the following statement is true or false:

**When you add any 2 even numbers, your answer is always even.**

Aysha’s answer

- \( a \) is any whole number.
- \( b \) is any whole number.
- \( 2a \) and \( 2b \) are any two even numbers.
- \( 2a + 2b = 2(a + b) \).

So Aysha says it’s true

Brian’s answer

- \( 2 + 2 = 4 \)
- \( 4 + 2 = 6 \)
- \( 2 + 4 = 6 \)
- \( 4 + 4 = 8 \)
- \( 2 + 6 = 8 \)
- \( 4 + 6 = 10 \)

So Brian says it’s true

Coby’s answer

Even numbers are numbers that can be divided by 2. When you add numbers with a common factor, 2 in this case, the answer will have the same common factor.

So Coby says it’s true

Deon’s answer

Even numbers end in 0, 2, 4, 6 or 8. When you add any two of these the answer will still end in 0, 2, 4, 6 or 8.

So Deon says it’s true

Eric’s answer

Let \( x \) = any whole number, \( y \) = any whole number.
- \( x + y = z \)
- \( z - x = y \)
- \( z - y = x \)
- \( z + z - (x + y) = x + y = 2z \)

So Eric says it’s true

Fiona’s answer

\[
\begin{array}{cccc}
\hline
\text{ } & \text{ } & \text{ } & \text{ } \\
\hline
\text{ } & \text{ } & \text{ } & \text{ } \\
\hline
\text{ } & \text{ } & \text{ } & \text{ } \\
\hline
\hline
\end{array}
\]

So Fiona says it’s true

a) Whose answer do you like best? ..............

b) Whose answer is closest to what you would do? ..............

c) Whose answer would get the best mark from your teacher? ..............
d) For each of the following, circle whether you agree, don’t know, or disagree.

The statement is:

**When you add any 2 even numbers, your answer is always even.**

<table>
<thead>
<tr>
<th>Statement</th>
<th>Agree</th>
<th>Don’t Know</th>
<th>Disagree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aysha’s answer...</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Brian’s answer...</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Coby’s answer...</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Deon’s answer...</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Eric’s answer...</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Fiona’s answer...</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

e) Suppose it has now been proved that:

**When you add any 2 even numbers, your answer is always even.**

Zoe asks what needs to be done to prove whether:

**When you add 2 even numbers that are square, your answer is always even.**

Tick (✓) either A or B.

(A) Zoe doesn’t need to do anything, the first statement has already proved this. ☐

(B) Zoe needs to construct a new proof. ☐
5 Prove whether the following statement is true or false. Write your answer in a way that would get you as good a mark as possible.

When you add any 2 odd numbers, your answer is always even.
6 Prove whether the following statement is true or false. Write your answer in a way that would get you as good a mark as possible.

If \( p \) and \( q \) are any two odd numbers, \((p + q) \times (p - q)\) is always a multiple of 4.
This diagram shows a triangle ABC. Side AB is the same length as side AC. Line BAE is straight.

a) Find the value of $c$ when $n = 140^\circ$.

Write down each step of your calculation.

b) Show that $c = \frac{1}{2}n$, whatever the value of $n$.

Write down all your steps.
c) In this diagram, lines SW, XT and VY are parallel. Line USV is straight.

Show that \(a = UST - b\).

Write down all your steps.

8 Squares A and B are identical. One corner of B is at the centre of A.

What fraction of A is overlapped by B?

......

Explain your answer.
Please go to the next question
Kath and Rose are thinking about the angles of this triangle.

They notice that two angles are ACUTE.

They notice that one angle is OBTUSE.

Kath says: If two angles of a triangle are ACUTE, the third angle is OBTUSE.

Rose says: If one angle of a triangle is OBTUSE, the other two angles are ACUTE.

a) Are Kath’s and Rose’s statements saying the same thing? . . . . . . . .

b) A triangle has an OBTUSE angle of 113.62°.

Suppose Rose is right.

Which one of these must also be right? Tick (✓) one box.

- You can be sure that the other two angles are both ACUTE.
- You can be sure that the other two angles are not both ACUTE.
- You can’t be sure whether the other two angles are both ACUTE until you know the size of both angles.

c) Is Kath’s statement true? . . . . . . .

Explain your answer.


d) Is Rose’s statement true? . . . . . . .

Explain your answer. 
Asim, Beth, Cara, Declan, Erin and Frank were trying to prove whether the following statement is true or false:

When you add the interior angles of any triangle, your answer is always 180°.

Asim’s answer
I tore the angles up and put them together.

It came to a straight line which is 180°. I tried for an equilateral and an isosceles as well and the same thing happened.

So Asim says it’s true

Beth’s answer
I drew an isosceles triangle, with c equal to 65°.

\[
\begin{align*}
\text{Statements} & \quad \text{Reasons} \\
a = 180° - 2c & \quad \text{... Base angles in isosceles triangle equal} \\
a = 50° & \quad 180° - 130° \\
b = 65° & \quad 180° - (a + c) \\
c = b & \quad \text{... Base angles in isosceles triangle equal} \\
\therefore a + b + c = 180°. \\
\end{align*}
\]

So Beth says it’s true

Cara’s answer
I drew a line parallel to the base of the triangle.

\[
\begin{align*}
\text{Statements} & \quad \text{Reasons} \\
p = s & \quad \text{... Alternate angles between two parallel lines are equal} \\
q = t & \quad \text{... Alternate angles between two parallel lines are equal} \\
p + q + r = 180° & \quad \text{... Angles on a straight line} \\
\therefore s + t + r = 180°. \\
\end{align*}
\]

So Cara says it’s true

Declan’s answer
I measured the angles of all sorts of triangles accurately and made a table.

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>34</td>
<td>36</td>
<td>180</td>
</tr>
<tr>
<td>95</td>
<td>43</td>
<td>42</td>
<td>180</td>
</tr>
<tr>
<td>35</td>
<td>72</td>
<td>73</td>
<td>180</td>
</tr>
</tbody>
</table>

They all added up to 180°.

So Declan says it’s true

Frank’s answer
I drew a tessellation of triangles and marked all the equal angles.

I know that the angles round a point add up to 360°.

So Frank says it’s true

a) Whose answer do you like best? ............

b) Whose answer is closest to what you would do? ............

c) Whose answer would get the best mark from your teacher? ............
d) For each of the following, circle whether you agree, don't know, or disagree.

The statement is:

**When you add the interior angles of any triangle, your answer is always 180°.**

<table>
<thead>
<tr>
<th>Agree</th>
<th>Don't Know</th>
<th>Disagree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asim's answer ...</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Beth's answer ...</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Cara's answer ...</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Declan's answer ...</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Erin's answer ...</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Frank's answer ...</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

e) Suppose it has now been proved that:

**When you add the interior angles of any triangle, your answer is always 180°.**

Zak asks what needs to be done to prove whether:

**When you add the interior angles of any right-angled triangle, your answer is always 180°.**

Tick (✓) either A or B.

(A) Zak doesn't need to do anything, the first statement has already proved this.  
(B) Zak needs to construct a new proof.
11 Prove whether the following statement is true or false. Write your answer in a way that would get you as good a mark as possible.

If you add the interior angles of any quadrilateral, your answer is always 360°.
A is the centre of a circle and AB is a radius. C is a point on the circumference where the perpendicular bisector of AB crosses the circle. Prove whether the following statement is true or false. Write your answer in a way that would get you as good a mark as possible.

Triangle ABC is always equilateral.
WAIT! Please go back to any questions you left out, then check all your answers. After that, if there is any time left over, please answer this questionnaire:

Z1  a) What did you feel about taking part in this survey?

b) Which question did you like best, and why?

c) Which question did you like least, and why?

d) Please add any other comments, if you wish to, about the survey.
Appendix B  Y10 Teacher Questionnaire

Teacher Questionnaire (Y10)

Name ..................................................

School ...............................................  LEA ......................

Name of your Y10 maths class with students involved in the survey ............

Please complete this questionnaire while your students are taking the proof survey.

Complete the details above and on pages 1 and 2, then work through the proof questions that follow.

Longitudinal Proof Project

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Please tick (✓) the appropriate boxes and complete the appropriate blanks

Female □ 1  Male □ 2

How many years teaching experience did you have at the start of this school year?  ....................

Your age:  under 25 □ 1  25 - 29 □ 2  30 - 39 □ 3  40 - 49 □ 4  50 - 59 □ 5  60 or more □ 6

School responsibility:
Head of maths □  Other (please specify)  .................................................................

Teaching Qualification:

Please specify type of qualification and subjects studied

<table>
<thead>
<tr>
<th>Type</th>
<th>Main subject (please specify)</th>
<th>Subsidiary subject (please specify)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree (if not BEd)</td>
<td>□</td>
<td>............................................</td>
</tr>
<tr>
<td>BEd</td>
<td>□</td>
<td>............................................</td>
</tr>
<tr>
<td>PGCE</td>
<td>□</td>
<td>............................................</td>
</tr>
<tr>
<td>Cert Ed</td>
<td>□</td>
<td>............................................</td>
</tr>
<tr>
<td>Other</td>
<td>□</td>
<td>............................................</td>
</tr>
</tbody>
</table>

Higher Education (apart from above):

Please specify type of qualification and main subject studied

<table>
<thead>
<tr>
<th>Type</th>
<th>Title (eg MEd)</th>
<th>Main subject</th>
<th>Year completed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Masters</td>
<td>□</td>
<td>.........................</td>
<td>.........................</td>
</tr>
<tr>
<td>PhD</td>
<td>□</td>
<td>.........................</td>
<td>.........................</td>
</tr>
</tbody>
</table>
Continuing Professional Development (CPD) or INSET in mathematics education

a. In this section do NOT include Government INSET for NNS or NOF, or any courses that you have mentioned in the Higher Education section, but DO include activities such as attending courses or conferences, writing text books, serving as an examiner, taking part in projects.

For the previous school year (2000 - 2001), estimate the number of sessions you were involved in CDP or INSET in mathematics education (where a session is a morning, afternoon, twilight or evening): ..........................

b. Current membership of a professional association:

ATM  MA  IMA  Other (please specify) .................................................................

Involvement in extra-curricular mathematics activities with students in your school during 2001 - 2002 (ie activities that are not part of the normal school mathematics curriculum, such as organising a mathematics club, organising students for master classes or UK Maths Challenge, taking students to mathematics events):

Yes  No  

Software that you have used this school year with Y10 students:

Logo  Dynamic geometry  Spreadsheet  Database  

Graphics calculator  Integrated learning system  SMILE software  The internet  

Other (please specify) ..................................................................................................................
A3 Aysha, Brian, Coby, Deon, Eric and Fiona were trying to prove whether the following statement is true or false:

**When you add any 2 even numbers, your answer is always even.**

a) Please take a minute to think how you would prove this, then go on to part b).

b) Consider Aysha, Brian, Coby, Deon, Eric and Fiona's answers on the next page.


   ii. Whose answer would your students say would get the best mark from you? . . .

   iii. Whose answer is closest to what you would do? . . .

c) Write a brief comment that might help these two students to move on:

Brian ..........................................................................................................................

..........................................................................................................................

..........................................................................................................................

..........................................................................................................................

Coby .........................................................................................................................

..........................................................................................................................

..........................................................................................................................

..........................................................................................................................
Aysha’s answer

\(a\) is any whole number.

\(b\) is any whole number.

\(2a\) and \(2b\) are any two even numbers.

\(2a + 2b = 2(a + b)\).

So Aysha says it’s true

Brian’s answer

\[2 + 2 = 4 \quad 4 + 2 = 6\]
\[2 + 4 = 6 \quad 4 + 4 = 8\]
\[2 + 6 = 8 \quad 4 + 6 = 10\]

So Brian says it’s true

Coby’s answer

Even numbers are numbers that can be divided by 2. When you add numbers with a common factor, 2 in this case, the answer will have the same common factor.

So Coby says it’s true

Deon’s answer

Even numbers end in 0, 2, 4, 6 or 8. When you add any two of these the answer will still end in 0, 2, 4, 6 or 8.

So Deon says it’s true

Eric’s answer

Let \(x\) = any whole number, \(y\) = any whole number.

\(x + y = z\)

\(z - x = y\)

\(z - y = x\)

\(z + z - (x + y) = x + y = 2z\)

So Eric says it’s true

Fiona’s answer

\[\begin{array}{c}
\text{\bullet\bullet\bullet} + \text{\bullet\bullet\bullet} \\
\text{\bullet\bullet\bullet\bullet\bullet\bullet} = \\
\text{\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet}\end{array}\]

So Fiona says it’s true
Asim, Beth, Cara, Declan, Erin and Frank were trying to prove whether the following statement is true or false:

**When you add the interior angles of any triangle, your answer is always 180°.**

a) Please take a minute to think how you would prove this, then go on to part b).

b) Consider Asim, Beth, Cara, Declan, Erin and Frank’s answers on the next page.


   ii. Whose answer would your students say would get the best mark from you? . . .

   iii. Whose answer is closest to what you would do? . . .

c) Write a brief comment that might help these two students to move on:

Asim ........................................................................................................................................

........................................................................................................................................

........................................................................................................................................

........................................................................................................................................

Declan ....................................................................................................................................

........................................................................................................................................

........................................................................................................................................

........................................................................................................................................
Proof Project June 2003

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Frank's answer
So Frank says it's true

I drew a tessellation of triangles and marked all the equal angles.

I know that the angles round a point add up to 360°.

So Frank says it's true

Asim's answer
I tore the angles up and put them together.

It came to a straight line which is 180°.

I tried for an equilateral and an isosceles as well and the same thing happened.

So Asim says it's true

Beth's answer
I drew an isosceles triangle, with c equal to 65°.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a = 180° − 2c</td>
<td>.... Base angles in isosceles triangle equal</td>
</tr>
<tr>
<td>a = 50°</td>
<td>180° − 130°</td>
</tr>
<tr>
<td>b = 65°</td>
<td>180° − (a + c)</td>
</tr>
<tr>
<td>c = b</td>
<td>Base angles in isosceles triangle equal</td>
</tr>
</tbody>
</table>

∴ a + b + c = 180°.

So Beth says it's true

Cara's answer
I drew a line parallel to the base of the triangle.

Statements Reasons
p = s  Alternate angles between two parallel lines are equal
q = t  Alternate angles between two parallel lines are equal
p + q + r = 180°... Angles on a straight line
∴ s + t + r = 180°.

So Cara says it's true

Declan's answer
I measured the angles of all sorts of triangles accurately and made a table.

They all added up to 180°.

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>34</td>
<td>36</td>
<td>180</td>
</tr>
<tr>
<td>95</td>
<td>43</td>
<td>42</td>
<td>180</td>
</tr>
<tr>
<td>35</td>
<td>72</td>
<td>73</td>
<td>180</td>
</tr>
<tr>
<td>10</td>
<td>27</td>
<td>143</td>
<td>180</td>
</tr>
</tbody>
</table>

So Declan says it's true

Erin's answer
If you walk all the way around the edge of the triangle, you end up facing the way you began. You must have turned a total of 360°.

You can see that each exterior angle when added to the interior angle must give 180° because they make a straight line.

This makes a total of 540°.

540° − 360° = 180°.

So Erin says it's true
Appendix C   Y10 School Questionnaire

Name of person completing questionnaire  ....................................

School .........................................       LEA    ...............  

Please complete this questionnaire at a convenient time and keep it with the other completed project materials. Complete the details above and overleaf.

Longitudinal Proof Project

Funded by the Economic and Social Research Council
**School data**

*Please tick the boxes which best describe your school*

<table>
<thead>
<tr>
<th>Selection</th>
<th>Single-sex/mixed</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>No academic selection</td>
<td>girls-only</td>
<td>urban</td>
</tr>
<tr>
<td>Some academic selection</td>
<td>boys-only</td>
<td>rural</td>
</tr>
<tr>
<td>Full academic selection</td>
<td>mixed-sex</td>
<td>suburban</td>
</tr>
</tbody>
</table>

**Year 10 data**

Approximate number of Y10 students in the school ..............................................

How are the current Y10 classes organised? *Please tick one box*

<table>
<thead>
<tr>
<th>Set</th>
<th>Banded</th>
<th>Mixed ability</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>☐ 1</td>
<td>☐ 2</td>
<td>☐ 3</td>
<td>☐ 4</td>
</tr>
</tbody>
</table>

For the Y10 students taking the Proof Survey, please give the approximate percentage of students from their class who you predict will be entered for the GCSE Higher tier ........................................

(If the students come from more than one class, please give the class name and percentage for each class)

**Mathematics curriculum data**

GCSE examination syllabus ...........................................................................................................

Main textbook / scheme in Year 10 ......................................................................................................

Total duration (in minutes) of Y10 mathematics lessons per week .......................................................

Is the department currently involved in the CAME project? Yes ☐ 1 No ☐ 2

**Extra-curricular mathematics activities**

Are any Y10 students involved (at school or elsewhere) in any mathematics activities that are not part of the normal school mathematics curriculum (eg maths club, master classes, UK Maths Challenge)?

Yes ☐ 1 No ☐ 2

If YES, please describe:
### Appendix D  Y10 Coding Scheme and Scores

**Coding Sheet Year 10 Proof Survey (and scores) combined version June 2003**

<table>
<thead>
<tr>
<th>Qu</th>
<th>Description of response and key points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td><strong>Generating data, spotting patterns, no structure</strong> Incorrect SCALAR (12) Incorrect FUNCT'L (13)</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Answer = 180 + no reason or unclear</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>+ incorrect scalar strategy (10 x 18)</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>+ uses incorrect functional str (3 x 60)</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>+ incorrect scalar and functional</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Some recognition of structure but incomplete or goes wrong or no reason or draws and counts</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>Answer = blank/wrong; partial structure (eg doubles but does not add 6), or generates correct data (eg, 6,18 7,20, 8,22) but stops or goes wrong</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>Answer = wrong: generates correct data (eg 10,26) but scales up (eg 10,26 x 6 gives 60,156)</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>Answer = wrong: sees correct structure initially but does not apply correctly, eg scales up inappropriately (eg 2 lots of 10 x 8, plus 1+1).</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>Answer = 126: sees white tiles as 10 rows of 6.</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>Answer = 126 + no reason or no clear reason or possibly false reason</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>Answer = 126 + erroneous use of (erroneous) table (eg sees white tiles in rows of 6, gets 6,18 and 16,20 and uses +2 x 2 to get g=2w+6).</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>Answer = 126 (or close to 126): eg, draws a photo-picture with 60 white tiles</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>Recognition and use of structure, specific</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>Answer = 126</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>Schematic diagram (not photo) or description of 60 white tiles surrounded by grey tiles. NOTHING MORE.</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>As code 30 but includes a table (or list) of data.</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>Recognition and use of structure, general, SCALAR</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>Answer = 126: sees that there are 10 times as many white tiles as in the given diagram, so will need 10 times as many grey tiles as there are above and below the given white tiles, plus the 3 tiles at each end: 10 x 12 + 6. <strong>No explicit naming of variables.</strong></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>Answer = 126: makes a table, using the idea that for every 6 extra white tiles that are 12 extra grey tiles. May go all the way to 60, 126.</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>Recognition and use of structure, general, FUNCTIONAL</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>Answer = 126</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>Shows procedure for getting the number of grey from the number of white: eg always two lots plus 6; double and add 6; 2 greys for each white; x 2 + 6; times 2 add 6. <strong>No explicit naming of variables</strong> (so eg does not use &quot;white&quot; to refer to total whites). Less emphasis on 60: concentrates on ops of x and +.</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>As code 42 but draws a table (or list) of data.</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>Towards Algebra: naming variables</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>Answer = 126: as code 42 but also names one or both variables in words (and may expresses general relationship between variables): eg, The number of grey is 2 times the number of white plus 6, or Double the amount of white tiles, add 6, or Double the white tiles, add 6, or white x 2 add 6.</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>As code 50, but uses a table (as in 30T, 41T, 42T).</td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>As code 50L, but uses a table.</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>No response 92 No time (or informative non response) 93 Miscellaneous</td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:** Do not penalise purely arithmetical errors (eg 2x60 = 100) but add E to the code

Do not penalise purely counting errors (eg code 22, 24) but add E to the code
<table>
<thead>
<tr>
<th>Qu</th>
<th>Description of response and key points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1b</td>
<td><strong>Passive description or pattern spotting</strong></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Answer = 6n or 60n (or 6n=18g, 60n=126g, etc). Add W, Y.</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>Answer = 3n. Add W, G, Y.</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Answer = 10n. Add W, G, Y.</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td><strong>Partial structure, or correct structure inadequately expressed (no letter or letter as object)</strong></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>Answer = 2n. Add W, G, Y.</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>Answer = 2n+3. Add W, G, Y.</td>
<td>1</td>
</tr>
<tr>
<td>30</td>
<td><strong>Correct structure, correctly expressed</strong></td>
<td></td>
</tr>
<tr>
<td>30W</td>
<td>Answer = 2w + 6</td>
<td></td>
</tr>
<tr>
<td>30Y</td>
<td>Answer = 2white + 6</td>
<td>2</td>
</tr>
<tr>
<td>91</td>
<td><strong>No response</strong></td>
<td></td>
</tr>
<tr>
<td>92</td>
<td><strong>No time or informative response</strong></td>
<td></td>
</tr>
<tr>
<td>93</td>
<td><strong>Miscellaneous</strong></td>
<td></td>
</tr>
</tbody>
</table>

Add W for w, G for g (except for code 30), Y for White. Add P for Power (in particular for n² + 6). Add B for correct ‘back to front’ expression.

Treat all letters other than w and g the same as n. Treat ‘grey’ the same as ‘g’.

---

<table>
<thead>
<tr>
<th>Qu</th>
<th>Description of response and key points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1b</td>
<td><strong>Passive description or pattern spotting</strong></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Answer = 6n or 60n (or 6n=18g, 60n=126g, etc).</td>
<td>0</td>
</tr>
<tr>
<td>11W</td>
<td>Answer = 6w or 60w, etc.</td>
<td></td>
</tr>
<tr>
<td>11Y</td>
<td>Answer = 6 white or 6 × white, etc.</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Answer = 3n.</td>
<td></td>
</tr>
<tr>
<td>12W</td>
<td>Answer = 3w.</td>
<td></td>
</tr>
<tr>
<td>12G</td>
<td>Answer = 3g.</td>
<td></td>
</tr>
<tr>
<td>12Y</td>
<td>Answer = 3 white, or 3 × white, etc.</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Answer = 10n.</td>
<td></td>
</tr>
<tr>
<td>13W</td>
<td>Answer = 10w.</td>
<td></td>
</tr>
<tr>
<td>13G</td>
<td>Answer = 10g.</td>
<td></td>
</tr>
<tr>
<td>13Y</td>
<td>Answer = 10 white, or 10 × white, etc.</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td><strong>Partial structure, or correct structure inadequately expressed (no letter or letter as object)</strong></td>
<td></td>
</tr>
<tr>
<td>21W</td>
<td>Answer = 2w.</td>
<td></td>
</tr>
<tr>
<td>21G</td>
<td>Answer = 2g.</td>
<td></td>
</tr>
<tr>
<td>21Y</td>
<td>Answer = 2 white or 2 × white, etc.</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>Answer = 2n+3.</td>
<td></td>
</tr>
<tr>
<td>22W</td>
<td>Answer = 2w+3.</td>
<td></td>
</tr>
<tr>
<td>22G</td>
<td>Answer = 2g+3 or 2n + 3g.</td>
<td></td>
</tr>
<tr>
<td>22Y</td>
<td>Answer = 2 × white + 3.</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>Answer = 2×2+6.</td>
<td></td>
</tr>
<tr>
<td>23G</td>
<td>Answer = 2g + 6, 2n + 6g.</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td><strong>Correct structure, correctly expressed</strong></td>
<td>2</td>
</tr>
<tr>
<td>30W</td>
<td>Answer = 2n + 6</td>
<td></td>
</tr>
<tr>
<td>30Y</td>
<td>Answer = 2white + 6</td>
<td></td>
</tr>
</tbody>
</table>
### L1c (Correct or incorrect) decision: no valid justification

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>Yes + nothing, or unclear* or vague (eg &quot;If you test it it makes sense&quot;); &quot;tried examples&quot;; repeats</td>
</tr>
<tr>
<td>13</td>
<td>No + nothing, or almost nothing *(but confused = 93, here and in what would otherwise be code 11)</td>
</tr>
<tr>
<td>14</td>
<td>No + example where condition does not hold, i.e sum not even (eg, 2+5=7 and 2×5=10, so product not odd; or &quot;even × odd is not odd&quot; (ie could be specific or general example)</td>
</tr>
</tbody>
</table>

### Part with incomplete or flawed justification

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>No + mixture of examples: condition does not hold (as in code 14) and valid counter example (as in code 31.32) (specific or general)</td>
</tr>
<tr>
<td>22</td>
<td>Yes + examples that confirm only, i.e only odd numbers (eg, 3+5=8 and/or 3×5=15) (spec or gen),</td>
</tr>
<tr>
<td>23</td>
<td>No + valid counter example (eg, 2,4) but incomplete (ie might consider sum but not product).</td>
</tr>
</tbody>
</table>

### (Basically) correct decision + correct justification

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>No + implicit counter example (eg, &quot;2+4=6 and 2×4=8&quot; or just &quot;2×4=8&quot;) but does say why example is important; may include non counter-examples that fit condition of sum=even, (eg, 1+3=4, 1×3=3)</td>
</tr>
<tr>
<td>32</td>
<td>No + explicit counter example (eg, [2+4=6 and 2×4=8, and 8 is even] ie states why the example is a counter example [Note: ignore examples which do not satisfy the condition (eg, 2,5 as in code 2) if it is clear that these are not meant to be part of the answer] (specific only)</td>
</tr>
</tbody>
</table>

### Correct decision + general justification/description in narrative form

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>No + If (A) the sum is even, then (B) the numbers could be both even, then (C) the product would be even B = code 41, A+B = code 42, B+C = code 43, A+B+C = code 44</td>
</tr>
<tr>
<td>+L</td>
<td>As codes 41 to 44 + algebraic description of the set of counter examples (eg E+E=E, E×E=E).</td>
</tr>
</tbody>
</table>

### Correct decision + general justification plus explanation of why justification is true

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>No + as code 4, and adds explanation of why two evens have an even sum and/or product.</td>
</tr>
<tr>
<td>50L</td>
<td>As code 50 but uses algebra (eg, 2x + 2y = 2[x+y], 2x×2y = 4xy). (use of E or O not sufficient here)</td>
</tr>
</tbody>
</table>

### L1d (Correct or incorrect) decision: no valid justification

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>Yes + anything, or nothing sensible, or vague (eg &quot;it always works&quot;).</td>
</tr>
<tr>
<td>13</td>
<td>Yes + repeat of statement (the product is odd, the sum is even)</td>
</tr>
<tr>
<td>14</td>
<td>True + false statement (specific or general).</td>
</tr>
</tbody>
</table>

### Correct Decision + incomplete or limited justification, but not false

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>Yes +confirmation by one empirical example</td>
</tr>
<tr>
<td>22</td>
<td>+confirmation by several empirical examples</td>
</tr>
<tr>
<td>23</td>
<td>+confirmation by examples + recognition that this is not enough</td>
</tr>
<tr>
<td>24</td>
<td>+crucial experiment i.e. random pair of odd numbers (eg, 19,23)</td>
</tr>
</tbody>
</table>

### Correct Decision + general justification of why numbers have to be odd + consequence

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Yes + If (A) the product is odd, then (B) the numbers could be both odd, then (C) the sum is even B = code 41, A+B = code 42, B+C = code 43, A+B+C = code 44</td>
</tr>
<tr>
<td>+L</td>
<td>As codes 41 to 44 + algebraic description, eg &quot;must be odd and O + O = E&quot;.</td>
</tr>
</tbody>
</table>

### Correct Decision + general justification plus explanation of why justification is true

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>Yes + As code 4 but adds explanation of why odd + odd is even (or why the numbers have to be odd)</td>
</tr>
<tr>
<td>50L</td>
<td>As code 50 but uses algebra (eg, 2x + 2y = 2[x+y], 2x×2y = 4xy). (use of E or O not sufficient here)</td>
</tr>
</tbody>
</table>

**Notes:**
- Codes 91, 92, 93 not listed for this question, but use them in the usual way.
- Note: Codes 91, 92, 93 not listed for this question, but use them in the usual way.
- Try not to penalise pure arithmetic errors (but code as 93 if they lead to confusion);
- Also, code as 93 if question misunderstood [eg if they have not grasped the meaning of Sum and Product, or if they focus on the numbers that fit part b), or if they borrow from "You can't be sure ... until you know what the numbers are"]; code 12 abolished: For "same as Fred's/Joe's" or "same as below/above", code the answer in the other box as if it had been written in the box you are currently coding, and add S for Same as Fred's/Joe's.
<table>
<thead>
<tr>
<th>Code</th>
<th>Description of response and key points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>Incorrect decision: confirming example or no explanation</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Answer = Yes + anything (including nothing)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>May have picture of quadrilateral where diagonals do cross at the centre</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Correct decision but no explanation</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Answer = No + nothing or not clear or not sensible.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(Include answers that question the trustworthiness of the sketch but which say nothing about the quadrilateral itself, eg &quot;Can't tell unless we use ruler and compass&quot; [but this may be given a higher code if there is a diagram]).</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Correct but only implicit reasons: weak explanation</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Answer = No</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ ambiguous or weak description of a counter example (or family of counter examples), including reference (without a diagram) to 'trapezium' or 'kite'.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>or</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ ambiguous or weak general explanation (global rather than analytic, ie concerned with the quadrilateral as a whole, rather than specifically with the endpoints of the diagonals), eg &quot;quadrilaterals with different sides aren't symmetrical&quot; &quot;quadrilaterals don't all have right angles&quot; &quot;If the sides are much smaller on one side, the diagonals won't cross in the middle&quot;.</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>Correct but only implicit reasons: weak diagram</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Answer = No + ambiguous or weak diagram (ie the quadrilateral looks almost like a rectangle and the centre is near but not at the intersection of the diagonals, and there are no constraints on the quadrilateral, as opposed to 31).</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>Correct and explicit counter example</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>Answer = No + decisive diagram but decisiveness not 'absolute', (ie does not show that it would never be possible for the diagonals to meet at the centre with such a picture); quadrilateral clearly not a rectangle; possible reference to 'trapezium' or 'kite'; accept drawing with one diagonal going through centre, but only if not rectangle-like.</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>Answer = No</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ clear description of counter example or absolutely decisive diagram(s) [but no dynamic argument (see code 41)].</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Code 32: Add D or N for Diagram or NoDiagram</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>Correct analytic reason</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Answer = No + use of dynamic argument, eg &quot;One of the points (R) may be slightly offset, so the diagonal (PR) no longer goes through the centre&quot;. Add D/N for Di/NoDi</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>Answer = No + clear general explanation (not dynamic=41 but nonetheless analytic, ie concerned with the endpoints of the diagonals rather than with the quadrilateral as a whole; but not simply a description of a counter example or family of counter examples), eg &quot;The corners could be anywhere and the diagonals will not necessarily go through centre&quot;. Add D/N for Di/NoDi, Add C for Cross (ie diagonals only drawn)</td>
<td>3</td>
</tr>
<tr>
<td>91</td>
<td>No response</td>
<td>0</td>
</tr>
<tr>
<td>92</td>
<td>Informative no response</td>
<td></td>
</tr>
<tr>
<td>93</td>
<td>Miscellaneous (includes: illegible answers; diagram that does not satisfy conditions, eg draws arrowhead, where one vertex not on circle; Yes and NO; neither Yes nor No)</td>
<td></td>
</tr>
</tbody>
</table>

For codes 32, 41, 42: letter D or N must be added
<table>
<thead>
<tr>
<th>G2b</th>
<th>Specific estimate, close but wrong</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td><strong>Answer</strong> = 1/3 or 1/5 (or decimal equivalent) + any or no explanation.</td>
</tr>
<tr>
<td>12</td>
<td><strong>Correct decision but no structural explanation</strong>&lt;br&gt;<strong>Answer</strong> = ¼ + no explanation, or perception (“it looks like a quarter”), or spurious reason (the overlapping sides are halved and half times half is a quarter”). <strong>Add ‘A’ for ‘always’.</strong></td>
</tr>
<tr>
<td>13</td>
<td><strong>Answer</strong> = ¼ + actual, valid measuring (eg draws grid and counts, or measures right angled triangle and calculates). <strong>Add E for answers that are close to but not exactly 1/4. Add ‘A’ for ‘always’.</strong></td>
</tr>
<tr>
<td>20</td>
<td><strong>Correct decision but only implicit reasons</strong>&lt;br&gt;<strong>Answer</strong> = ¼ + sensible but only partial explanation (if obviously not sensible, then code 12).&lt;br&gt;Could involve just one property (“corner is 90˚”) but might involve several properties, and/or valid operations (“90˚ is a quarter of 360˚”; “You can divide the square into 4”); might include some reference to <strong>turning</strong> (but not as for code 31 or 32). <strong>Add ‘A’ for ‘always’.</strong></td>
</tr>
<tr>
<td>31</td>
<td><strong>Correct decision relating to case where obviously 1/4</strong>&lt;br&gt;<strong>Answer</strong> = ¼ + refers to turning square D so that it is oriented as in one of these diagrams or draws one of the diagrams (eg turn it ‘to the side’ or ‘to the bottom’ or ‘till it is parallel’). <strong>Add ‘A’ for ‘always’. Add D/N for Di/NoDi</strong></td>
</tr>
<tr>
<td>32</td>
<td><strong>Answer</strong> = ¼ + claims that “the overlap fits 4 times”, by referring to turning square D through successive 90˚ turns, or to partitioning the square into 4 equal parts, as in the diagram; or draws diagram. <strong>Add D/N for Di/NoDi</strong></td>
</tr>
<tr>
<td>40</td>
<td><strong>Explanation of 1/4 in general case</strong>&lt;br&gt;<strong>Answer</strong> = ¼ + uses ‘compensation’ argument to explain why rotating from simple case (code 31) conserves the area of overlap (“on one side it is covering slightly more of the square and on the other the same amount less”). <strong>Add D/N for Di/NoDi</strong></td>
</tr>
<tr>
<td>91</td>
<td><strong>No response</strong></td>
</tr>
</tbody>
</table>

**Add ‘A’ for ‘always’ throughout**
<table>
<thead>
<tr>
<th>G4a</th>
<th>Description of response and key points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Calculating angle $u$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 Result of $360 - p'$ (normally &quot;40&quot;), somewhere on the page, with some evidence of where it came from.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3 &quot;40&quot;, somewhere on the page, but with no evidence of where it came from.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2 A value for $u$ other than 40, due to a factual error concerning 'angle at a point', eg &quot;angle at a point = 380&quot;&quot;, or to a method error (eg &quot;???&quot;), or to an unknown error (but if clearly due just to an arithmetic error, give the appropriate code and add E at the end).</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0 No discernible value for $u$ anywhere on the page.</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>Calculating $v + w$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 Result of $180 - u'$ (normally &quot;140&quot;), somewhere on the page, with some valid evidence of where it came from.</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3 Result of $180 - u'$ (normally &quot;140&quot;), somewhere on the page, but with no evidence of where it came from.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 A value for $v + w$ other than 140, due to a factual error concerning 'angle sum of a triangle'. eg &quot;angle sum = 360&quot;&quot;, or to a method error (eg &quot;???&quot;) or to an unknown error (but if clearly due just to an arithmetic error, give appropriate code and add E at the end).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 No discernible value for $v + w$.</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Calculating $v$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 Result of $\frac{v}{2}$ (normally &quot;70&quot;), somewhere on the page, with some valid evidence of where it came from.</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3 Result of $\frac{v}{2}$ (normally &quot;70&quot;), somewhere on the page, but with no evidence of where it came from.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 A value for $v$ other than 70, due to a factual error concerning 'base angles of an isosceles triangle', eg &quot;??????????&quot;, or to a method error (eg &quot;$v = u$&quot;) or to an unknown error (but if clearly due just to an arithmetic error, give the appropriate code and add E at the end).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 No discernible value for $v$.</td>
<td></td>
</tr>
<tr>
<td>91</td>
<td>No response at all</td>
<td></td>
</tr>
<tr>
<td>92</td>
<td>Informative no response</td>
<td></td>
</tr>
<tr>
<td>93</td>
<td>Miscellaneous</td>
<td></td>
</tr>
</tbody>
</table>

C and CE scores: 2

Where a viable parallel method is used, add P for Parallel method; try to fit to ABC coding, else code as 93P.
Add C for Correct to code if final answer is "70" (unless clearly obtained by wrong method).
Add CE if final answer is correct apart from arithmetic error.
### Version 01     15Dec02+

#### G4b
c=1/2n

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Empirical</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>One example (even if wrong, as long as it is numerical)</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>More than one example</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>Crucial experiment (value of n not ending in 0 or 5 and greater than 10)</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Exhaustive</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Analytic formal</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>naive algebraic</td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>1 correct algebraic expression (other than b=c)</td>
<td>0.5</td>
</tr>
<tr>
<td>52</td>
<td>2 correct algebraic expressions (other than b=c)</td>
<td>1</td>
</tr>
<tr>
<td>53</td>
<td>Correct derivation</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>don't use</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Narrative</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>no structure (ie 'narrative-naïve')</td>
<td>0</td>
</tr>
<tr>
<td>71</td>
<td>Expresses one correct relationship</td>
<td>0.5</td>
</tr>
<tr>
<td>72</td>
<td>Expresses two correct relationships</td>
<td>1</td>
</tr>
<tr>
<td>73</td>
<td>Correct derivation</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>Visual</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Usual 91, 92, 93</td>
<td></td>
</tr>
<tr>
<td>99</td>
<td>counter example</td>
<td>0</td>
</tr>
</tbody>
</table>

### Version 01     15Dec02+

#### G4c
c=1/2n

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Empirical</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>One example (c11,12,13: include pseudo example made to fit formula)</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>More than one example</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>Crucial experiment (value of n not ending in 0 or 5)</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Exhaustive</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Enactive</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Naïve (use 50, 70)</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Analytic formal</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>naive algebraic</td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>1 correct algebraic expression (other than a=f)</td>
<td>0.5</td>
</tr>
<tr>
<td>52</td>
<td>2 correct algebraic expressions (other than a=f)</td>
<td>1</td>
</tr>
<tr>
<td>53</td>
<td>Correct derivation</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>don't use</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Narrative</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>no structure (ie 'narrative-naïve')</td>
<td>0</td>
</tr>
<tr>
<td>71</td>
<td>Expresses one correct relationship</td>
<td>0.5</td>
</tr>
<tr>
<td>72</td>
<td>Expresses two correct relationships</td>
<td>1</td>
</tr>
<tr>
<td>73</td>
<td>Correct derivation</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>Visual</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Usual 91, 92, 93</td>
<td></td>
</tr>
<tr>
<td>99</td>
<td>counter example</td>
<td>0</td>
</tr>
<tr>
<td>Codes</td>
<td>91, 92, 93 not listed for this question, but use them in the usual way</td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>---------------------------------------------------------------------</td>
<td></td>
</tr>
</tbody>
</table>

### LG1

**Proof**

<table>
<thead>
<tr>
<th>(a) Yes</th>
<th>10 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes changed to No</td>
<td>31 1</td>
</tr>
<tr>
<td>No</td>
<td>32 2</td>
</tr>
</tbody>
</table>

### LG2

**Correct or incorrect decision: no valid justification**

<table>
<thead>
<tr>
<th>(b) both ACUTE</th>
<th>30 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>not both ACUTE</td>
<td>93 0</td>
</tr>
<tr>
<td>can't be sure</td>
<td>10 0</td>
</tr>
<tr>
<td>more than one</td>
<td>93 0</td>
</tr>
</tbody>
</table>

### LG3

**Decision with incomplete or flawed justification**

1. Yes + nothing, or unclear* or vague (eg "If you test it it makes sense"; "tried examples"; repeats) + [examples where condition does not hold (eg, ??)] + some examples that confirm and some that deny (eg, 30˚,40˚,110˚ and 60˚,60˚,60˚).

2. No + nothing, or almost nothing (eg "If acute + acute, then acute").

*(but confused = 93, here and in what would otherwise be code 11)*

3. No + example where condition does not hold, it does not start with two acute angles (eg "if 30˚ and 140˚, then third angle is 10˚ which is not obtuse") (could be specific or general example).

### LGc

**Correct decision + general justification/description in narrative form**

4. No + If (A) two angles are both <90˚, and (B) their sum ≥ 90˚, then (C) the third angle ≤ 90˚.

   B = code 41, A+B = code 42, B+C = code 43, A+B+C = code 44.

   (Only give c44 if there is a clear understanding that the sum of the two acute angles can be < 90˚.)

   (Give c32N for "No + right angled triangle has two acute angles", but only c23N for "No + right angled triangle").

   As codes 41 to 44 + algebraic description of the set of counter examples (eg if X+Y≥90, then Z<90).

   Add B, V (although B unlikely, and V more likely under c2 and c3 than here; don't assume that a simple drawing with unspecified angles is 'general').

   c4 answers involve a range of values, but this need be explicit in only one of the statements (A), (B), (C).

5. Code 5 redundant here. DO NOT USE. Covered by c4R.
### LGd

**Correct or incorrect decision: no valid justification**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>No</td>
<td>+ anything (including nothing) (unless c9), see below</td>
</tr>
<tr>
<td>13</td>
<td>Yes</td>
<td>+ nothing, or nothing sensible, or vague (eg &quot;it always works&quot;), + examples, some where condition not hold, ie first angle not obtuse (eg, right triangle) (spec or gen) + repeat of statement (&quot;If 1 obtuse, then 2 acute&quot;), or repeat of statement back to front + false statement (specific or general) (unless c93, see below)</td>
</tr>
</tbody>
</table>

**Correct Decision + incomplete or limited justification, but not false**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>Yes</td>
<td>+ confirmation by one empirical example</td>
</tr>
<tr>
<td>22</td>
<td></td>
<td>+ confirmation by several empirical examples</td>
</tr>
<tr>
<td>23</td>
<td></td>
<td>+ confirmation by examples + recognition that this is not enough</td>
</tr>
<tr>
<td>24</td>
<td></td>
<td>+ crucial experiment i.e. 'random' angle (eg 147˚) [not ending in 0 or 5, not 90-95˚, not 113˚].</td>
</tr>
</tbody>
</table>

**Correct Decision + counter-counter example (proof by contradiction)**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>Yes</td>
<td>+ argument that there can't be another obtuse angle (as angle sum would be &gt; 180˚) (but &quot;so other angles must be acute&quot; left implicit).</td>
</tr>
<tr>
<td>32</td>
<td>Yes</td>
<td>+ argument that there can't be another obtuse angle (as angle sum would be &gt; 180˚) (and &quot;so other angles must be acute&quot; made explicit).</td>
</tr>
</tbody>
</table>

**Correct Decision + general justification of why resulting angles have to be acute:**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Yes</td>
<td>+ If (A) one angle &gt; 90˚, then (B) the sum of the other two &lt; 90, so (C) they are both acute</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B = code 41, A+B+O = code 42, B+O = code 43, A+B+O = code 44.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Give c13 or c93 for 'Yes+any mention of &quot;right angled triangle&quot;').</td>
</tr>
<tr>
<td></td>
<td>As codes 41 to 44 + algebraic description, eg &quot;eg if Z&gt;90, then X+Y&lt;90&quot;.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Add B (though B unlikely here)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Add V (though V unlikely here)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Add M (redundant here; only allow a minimal value in c4 if it is stated that it is a minimal value)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c4 answers involve a range of values, but this need be explicit in only one of the statements (A), (B), (C).</td>
<td></td>
</tr>
</tbody>
</table>

**Correct Decision + general justification plus explanation of why justification is true**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td></td>
<td>Code 5 redundant here. DO NOT USE. Covered by c4R.</td>
</tr>
</tbody>
</table>

---

Parts c) and d):

- try not to penalise pure arithmetic errors (but code as 93 if they lead to confusion);
- also, code as 93 if question misunderstood [eg if they have not grasped the meaning of Acute and Obtuse, or if they think the angle sum is 360˚, say, or if they focus on values that fit part b) (113˚), or if they borrow from "You can't be sure ... until you know what the angles are"]
- code 12 abolished: For "same as Rose/s/ Kath/s" or "same as below/above", code the answer in the other box as if it had been written in the box you are currently coding, and add S (for L1c) or S (for L1d) to the code.
<table>
<thead>
<tr>
<th>Code</th>
<th>Description of ODDS</th>
<th>Adding Odds</th>
</tr>
</thead>
<tbody>
<tr>
<td>HA4</td>
<td>Description of ODDS</td>
<td>Adding Odds</td>
</tr>
<tr>
<td>code 1</td>
<td>Empirical</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>One example</td>
<td>0 = no calculation (is this likely)</td>
</tr>
<tr>
<td>12</td>
<td>More than one example</td>
<td>1 = just does calculation(s)</td>
</tr>
<tr>
<td>13</td>
<td>Crucial experiment (at least one number &gt; 10 and 'random')</td>
<td>2 = '1' and states the result is even Add E for Exhaustive (or systematic) Add T for 2-wayTable format</td>
</tr>
<tr>
<td>code 2</td>
<td>Exhaustive</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Odd numbers end in 1, 3, 5, 7, 9</td>
<td>+0 = nothing new</td>
</tr>
<tr>
<td>22</td>
<td>moderately successful attempt at exhaustive list of end-digit calculations (eg incomplete but systematic)</td>
<td>+1 = &quot;result of adding end digits is even&quot;, or &quot;result is 0, 2, 4, 6, 8&quot; or list of actual results +1 = reference to the result being the 'end digit(s)' +1 = Line 3 + &quot;and so the answer is even&quot;</td>
</tr>
<tr>
<td>23</td>
<td>impressive attempt at exhaustive list of end-digit calculations</td>
<td>(So to get a '3' here, need to be close to saying &quot;the end digit will be even, which means the answer will be even&quot;)</td>
</tr>
<tr>
<td>code 3</td>
<td>[Enactive]</td>
<td></td>
</tr>
<tr>
<td>code 4</td>
<td>[Naïve]</td>
<td></td>
</tr>
<tr>
<td>code 5</td>
<td>Analytic formal (correct)</td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>2a+1</td>
<td>1 = write an expression for the sum, or just for the end bit, ie 1+1=2</td>
</tr>
<tr>
<td>52</td>
<td>2a+1, 2b+1</td>
<td>2 = produces expression of the form 2 x A, where A is an expression, or perhaps just a single letter OR a statement that expression consists of a sum of several EVEN numbers 3 = Line 2 + &quot;so the answer is even&quot; (So to get a '3' need to be close to saying &quot;2 x A is even&quot; OR close to including &quot;Even+Even is Even&quot;) Add A for pureAlgebra</td>
</tr>
<tr>
<td>code 6</td>
<td>Analytic semi-formal</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>use of letter(s) but no structure (eg a = odd)</td>
<td>1 = write an expression for the sum, or just for the end bit, ie 1+1=2</td>
</tr>
<tr>
<td>61</td>
<td>a is even, a+1 is odd</td>
<td>2 = produces expression of the form 2 x A, where A is an expression, or perhaps just a single letter OR a statement that expression consists of a sum of several EVEN numbers 3 = Line 2 + &quot;so the answer is even&quot; (So to get a '3' need to be close to including &quot;Even+Even is Even&quot;) Add A for pureAlgebra</td>
</tr>
<tr>
<td>62</td>
<td>a and b are even, a+1, b+1 are odd</td>
<td></td>
</tr>
<tr>
<td>code 7</td>
<td>(Analytic) narrative</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>no structure (ie 'narrative-naïve')</td>
<td>0 = nothing more</td>
</tr>
<tr>
<td>71</td>
<td>Partial structure (eg general idea that odds go up in 2s)</td>
<td>1 = extra 1 + extra 1 = 2 or even 2 = so the result consists of a sum of several EVEN numbers 3 = '2' + and so the answer is even Add G for Generic example</td>
</tr>
<tr>
<td>72</td>
<td>ODD is EVEN+1 (general explanation)</td>
<td></td>
</tr>
<tr>
<td>code 8</td>
<td>Visual</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>no structure shown for individual odd numbers</td>
<td>1 = draws the result as a '2 by something' rectangle of dots +1 = indicates the structure of the result in some way (interlocking shapes, or the two 'odd' dots highlighted, or possibly referred to verbally)</td>
</tr>
<tr>
<td>81</td>
<td>partial structure (eg number-line showing odds go up in 2s)</td>
<td>+1 = and so the answer is even Add G for General 'example' Add V for pureVisual</td>
</tr>
<tr>
<td>82</td>
<td>Oddness Structure shown (generic example)</td>
<td></td>
</tr>
<tr>
<td>code 9</td>
<td>Usual 91, 92, 93</td>
<td></td>
</tr>
<tr>
<td>99</td>
<td>counter example</td>
<td></td>
</tr>
</tbody>
</table>

Where students used mixed styles of answer, choose the dominant style or, if they can noy be separated out, the one that produces the better proof. Code the minor style (if there is one) in a separate column using just the first two digits of the usual 3-digit code.
<table>
<thead>
<tr>
<th>Code</th>
<th>Description of ((p+q)) and ((p-q))</th>
<th>Multiplying ((p+q)) by ((p-q))</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>One example</td>
<td>0 = no calculation (is this likely)</td>
</tr>
<tr>
<td>12</td>
<td>More than one example</td>
<td>1 = just does calculation(s) 2 = '1' and states &quot;the result is an M4&quot; (or &quot;is true&quot;) 3 = demonstrates that the result is an M4 Add G for empirical Generalisation (eg &quot;one bracket is always an M4&quot;)</td>
</tr>
<tr>
<td>13</td>
<td>Crucial experiment (at least one number &gt; 10 and 'random')</td>
<td></td>
</tr>
</tbody>
</table>

**Code 2: Exhaustive**

- 21 - This category will not apply. Students might produce an exhaustive argument to show, rather than just state, that \((p+q)\) and \((p-q)\) are even, but we are not interested in this 'history'.
- 22 -
- 23 -

**Code 3: [Enactive]**

- 31 -
- 32 -
- 33 -

**Code 4: [Naïve]**

- 41 -
- 42 -

**Code 5: Analytic formal (correct)**

- 51 - 2a+1
- 52 - 2a+1, 2b+1

1 = write an expression for the product
2 = produces expression of the form \(4 \times A\), where \(A\) is an expression, or perhaps just a single letter OR a statement that "the expression consists of the product of several EVEN numbers (or the sum of several M4s)"
3 = Line 2 + "so the answer is an M4" (So to get a '3' need to be close to saying "4 \times A is an M4" OR close to saying "M2\times M2 = M4")
Add A for pureAlgebra
Add M for Multiplies brackets (eg \(p^2 - q^2\))

**Code 6: Analytic semi-formal**

- 60 - use of letter(s) but no structure (eg \(a = \text{odd}\))
- 61 - \(a\) is even, \(a+1\) is odd
- 62 - \(a\) and \(b\) are even, \(a+1\), \(b+1\) are odd

1 = write an expression for the product
2 = produces expression of the form \(4 \times A\), where \(A\) is an expression, or perhaps just a single letter OR a statement that "the expression consists of the product of several EVEN numbers (or the sum of several M4s)"
3 = Line 2 + "so the answer is an M4" (So to get a '3' need to be close to saying "4 \times A is an M4" OR close to saying "M2\times M2 = M4")
Add A for pureAlgebra
Add M for Multiplies brackets (eg \(p^2 - q^2\))

**Code 7: (Analytic) narrative**

<table>
<thead>
<tr>
<th>Code</th>
<th>(Analytic) narrative</th>
<th>(p^2 - q^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>no structure (ie 'narrative-naïve')</td>
<td>0 = nothing more</td>
</tr>
<tr>
<td>71</td>
<td>Partial structure (eg states that one of ((p+q)) or ((p-q)) is even; or that (p=E+1))</td>
<td>0 = nothing more</td>
</tr>
<tr>
<td>72</td>
<td>((p+q)) and ((p-q)) are both even (or (p'=\text{Odd}, q'=\text{Odd})) (general explanation)</td>
<td>Add M for Multiplies brackets (eg (p^2 - q^2))</td>
</tr>
</tbody>
</table>

**Code 8: [Visual]**

- 80 - no structure shown for \((p+q)\) and \((p-q)\)
- 81 - partial structure (eg structure shown for one bracket)
- 82 - Evenness Structure shown for \((p+q)\) and \((p-q)\) (generic example)

1 = draws the result as a '4 by something' rectangle of dots
+1 = indicates the structure of the result in some way (interlocking shapes, or the two 'odd' dots highlighted, or possibly referred to verbally) (CAN THIS HAPPEN??)
+1 = and so the answer is even (CAN THIS HAPPEN??)
Add G for General example (CAN THIS HAPPEN??)
Add V for pureVisual (CAN THIS HAPPEN??)

**Code 9: Usual**

<table>
<thead>
<tr>
<th>Code</th>
<th>Usual</th>
</tr>
</thead>
<tbody>
<tr>
<td>91</td>
<td>92, 93</td>
</tr>
</tbody>
</table>

Where students used mixed styles of answer, choose the dominant style or, if they can be separated out, the one that produces the better proof. Code the minor style (if there is one) in a separate column using just the first two digits of the usual 3-digit code.
<table>
<thead>
<tr>
<th>HG4</th>
<th>Description of quadrilateral</th>
<th>Argument used</th>
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</thead>
<tbody>
<tr>
<td>code 1</td>
<td><strong>Empirical</strong></td>
<td></td>
</tr>
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</table>
| 11 | Rectangle or square (one or more) | 0 = nothing, or bald statement that \(\Sigma=360^\circ\). \(1 = \text{\textquoteleft\textquoteleft demonstrates\textquoteright\textquoteright\textquoteleft\ that }\Sigma=360^\circ \text{ for chosen quadrilateral (eg a 90,90,90,90 square or a quad whose marked angles sum to 360).}
\(2 = \text{\textquoteleft\textquoteleft expresses/uses an incorrect relationship (eg opposite angles of parallelogram=180\(^\circ\)\textquoteright\textquoteright\).}
\(3 = \text{\textquoteleft\textquoteleft expresses/uses a correct relationship (eg opposite angles of cyclic quadrilateral=180\(^\circ\)\textquoteright\textquoteright\) but does not get to }\Sigma=360^\circ \text{ (eg next step incorrect).}
\(4 = \text{\textquoteleft\textquoteleft uses a correct relationship to get to }\Sigma=360^\circ \text{ for chosen quadrilateral.}
Add X for Exterior angles, P for Parallel construction line(s), T for Triangles, N for NO diagram, D for Dynamic, B for Backwards (circular) |
| code 2 | **[Exhaustive]** | |
| code 3 | **Enactive** | |
| 31 | Rectangle or square (one or more) | 0 = baldly states angles make 360\(^\circ\).
\(1 = \text{\textquoteleft\textquoteleft produces a drawing which shows four angle making a complete circle}
\(2 = \text{\textquoteleft\textquoteleft size of angles or labels (but not both) match angles in original quadrilateral}
\(3 = \text{\textquoteleft\textquoteleft for completely general} |
| code 4 | **[Na\'ive]** | |
| code 5 | **Analytic formal** (correct or not) | Uses a, b, c, d, say, for angles (in exposition, not just in diag)
\(0 = \text{nothing, or bald statement that }\Sigma=360^\circ.
\(1 = \text{\textquoteleft\textquoteleft demonstrates\textquoteright\textquoteleft that }\Sigma=360^\circ \text{ for chosen quadrilateral (not sure this is possible).}
\(2 = \text{\textquoteleft\textquoteleft expresses/uses an incorrect relationship (eg opposite angles of parallelogram=180\(^\circ\)\textquoteright\textquoteright\).
\(3 = \text{\textquoteleft\textquoteleft expresses/uses a correct relationship (eg opposite angles of cyclic quadrilateral=180\(^\circ\)\textquoteright\textquoteright\) but does not get to }\Sigma=360^\circ \text{ (eg next step incorrect).}
\(4 = \text{\textquoteleft\textquoteleft uses a correct general relationship to get to }\Sigma=360^\circ \text{ for completely general quadrilateral}
Add X for Exterior angles, P for Parallel construction line(s), T for Triangles (ie \(\Sigma+\Sigma=180+180=360\)) Add N for NO diagram, F for Formula \(\text{(NO A for pureAlg)}
| code 6 | **[Analytic semi-formal]** (use c5) | |
| code 7 | **(Analytic) narrative** | Uses get use a, b, c, d, say, for angles (in exposition, whether or not in diag)
\(0 = \text{nothing, or bald statement that }\Sigma=360^\circ.
\(1 = \text{\textquoteleft\textquoteleft produces a drawing which shows four angle making a complete circle}
\(2 = \text{\textquoteleft\textquoteleft size of angles or labels (but not both) match angles in original quadrilateral}
\(3 = \text{\textquoteleft\textquoteleft for completely general} |
| code 8 | **Visual** | |
| 80 | ? | Uses get use a, b, c, d, say, for angles (in exposition, whether or not in diag)
\(0 = \text{nothing, or bald statement that }\Sigma=360^\circ.
\(1 = \text{\textquoteleft\textquoteleft produces a drawing which shows four angle making a complete circle}
\(2 = \text{\textquoteleft\textquoteleft size of angles or labels (but not both) match angles in original quadrilateral}
\(3 = \text{\textquoteleft\textquoteleft for completely general} |
| code 9 | **Usual** 91, 92, 93 | Generally, code a mistake that leads to \textquoteleft\textquoteleft No\textquoteright\textquoteright\) as c93 |

Where students used mixed styles of answer, choose the dominant style or, if they can be separated out, the one that produces the better proof. Code the minor style (if there is one) in a separate column using just the first two digits of the usual 3-digit code.
<table>
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<tr>
<th>HG7</th>
<th>Description of triangle (Properties)</th>
<th>Argument used</th>
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<tr>
<td>code 1</td>
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<tr>
<td>11</td>
<td>Direct properties based on measurement</td>
<td>0 = nothing, or bald statement that ( \Delta = )Equilateral.</td>
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<td>12</td>
<td>[-]</td>
<td>1 = 'demonstrates' that ( \Delta = )Equilateral by measuring 3 sides or 3 angles. (Note: Have not used this well or consistently: usually when they've drawn a confusing example have called it 700 rather than 110, say)</td>
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<tr>
<td>13</td>
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<tr>
<td>code 4</td>
<td>[Naïve]</td>
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<tr>
<td>code 5</td>
<td>Analytic formal (correct or not)</td>
<td>Dominant style is 'AB=AC' rather than 'AB, AC same length'</td>
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<tr>
<td>51</td>
<td>some 'direct properties' (eg AB=AC, AD=DB, D=90, ( \Delta ADC=\Delta BDC )) but no reasons</td>
<td>0 = nothing, or bald statement that ( \Delta = )Equilateral.</td>
</tr>
<tr>
<td>52</td>
<td>some properties, some valid reasons (eg 'radii', 'bisector', 'perp')</td>
<td>1 = some 'derived' properties (CA=CB or AB=AC=CB, or equivalent) but no reason or explicit deduction</td>
</tr>
<tr>
<td>53</td>
<td>all properties needed for particular proof (eg AB=AC, AD=DB, D=90, ( \Delta ADC=\Delta BDC )), all with valid reasons</td>
<td>2 = &quot;1&quot; + some reason or deduction</td>
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<tr>
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<td>Analytic semi-formal (Use c5 instead)</td>
<td>3 = complete properties+deductions (ie CA=CB and ( \Delta = )Equilateral)</td>
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<tr>
<td>code 7</td>
<td>(Analytic) narrative</td>
<td>May use some of 'AB=AC' but dominated by long thread of text</td>
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<td>70</td>
<td>no structure (ie 'narrative-naïve')</td>
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<td>0 = nothing, or bald statement that ( \Delta = )Equilateral (include explicit properties, if no reasons given)</td>
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<td>some properties, some valid reasons (eg 'radii', 'bisector', 'perp')</td>
<td>1 = some 'derived' properties (CA=CB or AB=AC=CB, or equivalent) but no reason or explicit or valid deduction</td>
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<td>73</td>
<td>all properties needed for particular proof (eg AB=AC, AD=DB, D=90, ( \Delta ADC=\Delta BDC )), all with valid reasons</td>
<td>2 = &quot;1&quot; + some valid reason or deduction</td>
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<tr>
<td>code 8</td>
<td>(Visual)</td>
<td>3 = complete properties+deductions (ie CA=CB and ( \Delta = )Equilateral)</td>
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<tr>
<td>code 9</td>
<td>Usual 91, 92, 93</td>
<td>Add S for use of Symmetry, C for use of Congruence, P for Pythagoras (NO A for pureAlg)</td>
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</table>

| c99 | counter example | Generally, code a mistake that leads to 'No' as c93 |

Where students used mixed styles of answer, choose the dominant style or, if they can be separated out, the one that produces the better proof. Code the minor style (if there is one) in a separate column using just the first two digits of the usual 3-digit code.
Scores for HA4, HA7, HG4, HG7

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Proof Project  176  June 2003