Appendix A Y10 Proof Survey

INSTITUTE OF EDUCATION
UNIVERSITY OF LONDON

Year 10 Proof Survey

Name .... Maths Class ....
first name surname

School .... Boy or Girl ....

Date of Birth .... Today's Date ....
day month year day month year

You have 55 minutes to answer these questions.

In two of the questions you will be asked to choose from a range of answers.

In all the other questions, you will be asked to produce your own answers. We are interested in your thinking as well as your answers, so please show all your rough working for these questions.

Put your rough working on the same page as your answer; use the answer box or any spare space on the page.

In most questions you will be asked for explanations. Make these as clear as you can, but don't make them longer than necessary.

Use a pen. You may cross things out, but do not rub out any of your work and do not use correction fluid.

Do not use a calculator.

The questions are not ordered by difficulty. If you get stuck on a question, don't worry - leave it till later.

On the last page there is a questionnaire. Only fill this in if you have done all you can on the other questions and there is time left over.

A

Sch

Cla

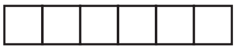
Stu

Longitudinal Proof Project

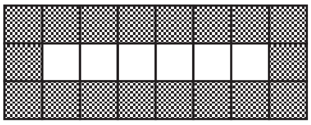
Funded by the Economic and Social Research Council

1 Lisa has some white square tiles and some grey square tiles.
They are all the same size.

She makes a row
of white tiles.



She surrounds the white
tiles by a single layer
of grey tiles.



a) How many grey tiles does she need to surround a row of 60 white tiles? . . . .

Show how you obtained your answer.

b) Write an expression for the number of grey tiles
needed to surround a row of n white tiles. . . . .

Please leave blank

A1

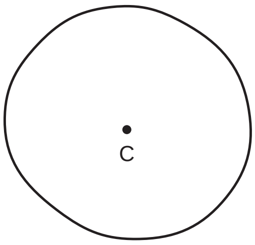
2 Vincent sketches a circle. He calls the centre C.

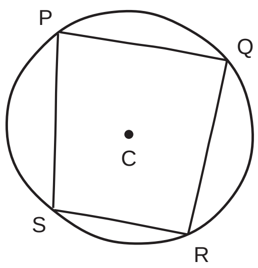
He then draws a quadrilateral PQRS, whose corners lie on the circle.

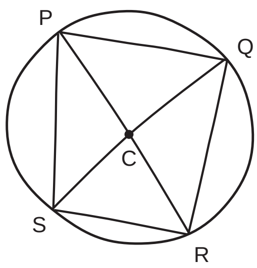
He then draws the diagonals of the quadrilateral.

Please leave blank

G1







Vincent says
"Whatever quadrilateral I draw with corners on a circle,
the diagonals will always cross at the centre of the circle".

Is Vincent right? ....

Explain your answer.

Please leave blank

LA1

3 Joe and Fred are thinking about the pair of numbers 5 and 9.

They notice that the SUM $(5 + 9)$ is EVEN.

They notice that the PRODUCT (5×9) is ODD.

Joe says: If the SUM of two whole numbers is EVEN, their PRODUCT is ODD.

Fred says: If the PRODUCT of two whole numbers is ODD, their SUM is EVEN.

a) Are Joe's and Fred's statements saying the same thing? . . . .

b) The PRODUCT of two whole numbers is 1247.

Suppose Fred is right.

Which one of these must also be right? Tick (✓) one box.

❑ You can be sure that the SUM of the two numbers is EVEN.

❑ You can be sure that the SUM of the two numbers is ODD.

❑ You can't be sure whether the SUM is ODD or EVEN until you know what the two numbers are.

c) Is Joe's statement true? . . . .

Explain your answer.

d) Is Fred's statement true? . . . .

Explain your answer.

4 Aysha, Brian, Coby, Deon, Eric and Fiona were trying to prove whether the following statement is true or false:

Please leave blank

A3

When you add any 2 even numbers, your answer is always even.

Aysha's answer

a is any whole number.

b is any whole number.

$2a$ and $2b$ are any two even numbers.

$2a + 2b = 2(a + b)$.

So Aysha says it's true

Brian's answer

$2 + 2 = 4$	$4 + 2 = 6$
$2 + 4 = 6$	$4 + 4 = 8$
$2 + 6 = 8$	$4 + 6 = 10$

So Brian says it's true

Coby's answer

Even numbers are numbers that can be divided by 2. When you add numbers with a common factor, 2 in this case, the answer will have the same common factor.

So Coby says it's true

Deon's answer

Even numbers end in 0, 2, 4, 6 or 8. When you add any two of these the answer will still end in 0, 2, 4, 6 or 8.

So Deon says it's true

Eric's answer

Let x = any whole number, y = any whole number.

$x + y = z$

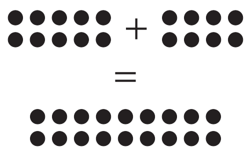
$z - x = y$

$z - y = x$

$z + z - (x + y) = x + y = 2z$

So Eric says it's true

Fiona's answer



So Fiona says it's true

a) Whose answer do you like best? ....

b) Whose answer is closest to what you would do? ....

c) Whose answer would get the best mark from your teacher? ....

4 *Continued*

d) For each of the following, circle whether you agree, don't know, or disagree.

The statement is:

When you add any 2 even numbers, your answer is always even.

	agree	don't know	disagree
Aysha's answer ... shows you that the statement is **always true**	1	2	3
Brian's answer ... shows you that the statement is **always true**	1	2	3
Coby's answer ... shows you that the statement is **always true**	1	2	3
Deon's answer ... shows you that the statement is **always true**	1	2	3
Eric's answer ... shows you that the statement is **always true**	1	2	3
Fiona's answer ... shows you that the statement is **always true**	1	2	3

Please leave blank

HA2

e) Suppose it has now been proved that:

When you add any 2 even numbers, your answer is always even.

Zoe asks what needs to be done to prove whether:

When you add 2 even numbers that are square, your answer is always even.

Tick (✓) either A or B.

(A) Zoe doesn't need to do anything, the first statement has already proved this. ❑

(B) Zoe needs to construct a new proof. ❑

5 Prove whether the following statement is true or false. Write your answer in a way that would get you as good a mark as possible.

When you add any 2 odd numbers, your answer is always even.

Please leave blank

HA4

6 Prove whether the following statement is true or false. Write your answer in a way that would get you as good a mark as possible.

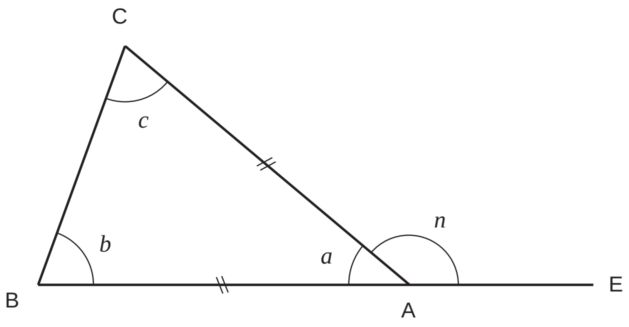
Please leave blank

HA7

If p and q are any two odd numbers, $(p+q) \times (p-q)$ is always a multiple of 4.

7

This diagram shows
a triangle ABC.
Side AB is the same
length as side AC.
Line BAE is straight.



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G4

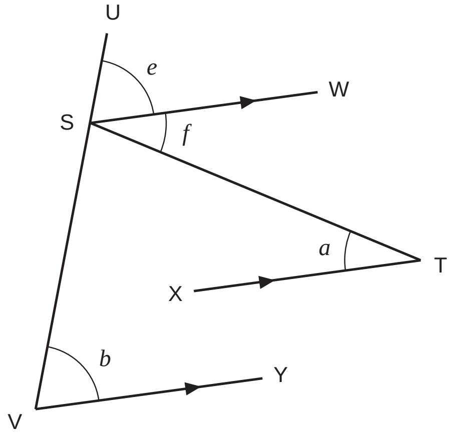
a) Find the value of c when $n = 140^\circ$. . . . .

Write down each step of your calculation.

b) Show that $c = \frac{1}{2}n$, whatever the value of n.

Write down all your steps.

c) In this diagram, lines SW, XT and VY are parallel. Line USV is straight.

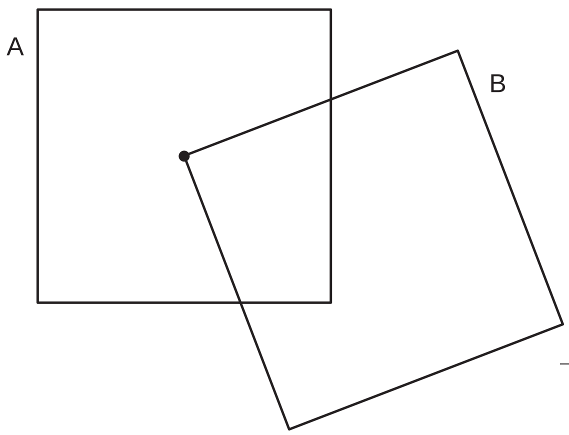


Show that $a = \hat{\text{UST}} - b$.

Write down all your steps.

Please leave blank

8 Squares A and B are identical. One corner of B is at the centre of A.



What fraction of A is overlapped by B ?

. . . .

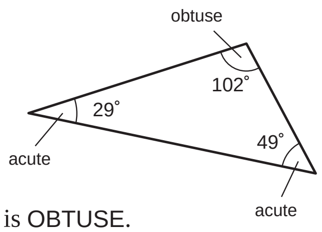
Explain your answer.

G2b

Please
leave
blank

Please go to the next question

9 Kath and Rose are thinking about the angles of this triangle.



They notice that two angles are ACUTE.

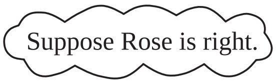
They notice that one angle is OBTUSE.

Kath says: If two angles of a triangle are ACUTE, the third angle is OBTUSE.

Rose says: If one angle of a triangle is OBTUSE, the other two angles are ACUTE.

a) Are Kath's and Rose's statements saying the same thing? . . . .

b) A triangle has an OBTUSE angle of 113.62˚.



Which one of these must also be right? Tick (✓) one box.

❏ You can be sure that the other two angles are both ACUTE.

❏ You can be sure that the other two angles are not both ACUTE.

❏ You can't be sure whether the other two angles are both ACUTE until you know the size of both angles.

c) Is Kath's statement true? . . . .

Explain your answer.

d) Is Rose's statement true? . . . .

Explain your answer.

Please leave blank

LG1

10 Asim, Beth, Cara, Declan, Erin and Frank were trying to prove whether the following statement is true or false:

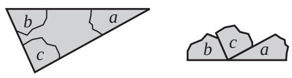
When you add the interior angles of any triangle, your answer is always 180˚.

Please leave blank

G3

Asim's answer

I tore the angles up and put them together.

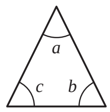


It came to a straight line which is 180˚.
I tried for an equilateral and an isosceles as well and the same thing happened.

So Asim says it's true

Beth's answer

I drew an isosceles triangle, with *c* equal to 65˚.

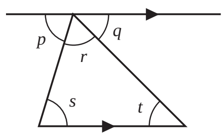


Statements	*Reasons*
$a = 180˚ - 2c$	Base angles in isosceles triangle equal
$a = 50˚$	$180˚ - 130˚$
$b = 65˚$	$180˚ - (a + c)$
$c = b$	Base angles in isosceles triangle equal
$\therefore\ a + b + c = 180˚$.	

So Beth says it's true

Cara's answer

I drew a line parallel to the base of the triangle.



Statements	*Reasons*
$p = s$	Alternate angles between two parallel lines are equal
$q = t$	Alternate angles between two parallel lines are equal
$p + q + r = 180˚$...	Angles on a straight line
$\therefore\ s + t + r = 180˚$.	

So Cara says it's true

Declan's answer

I measured the angles of all sorts of triangles accurately and made a table.

a	*b*	*c*	*total*
110	34	36	180
95	43	42	180
35	72	73	180
10	27	143	180

They all added up to 180˚.

So Declan says it's true

Erin's answer

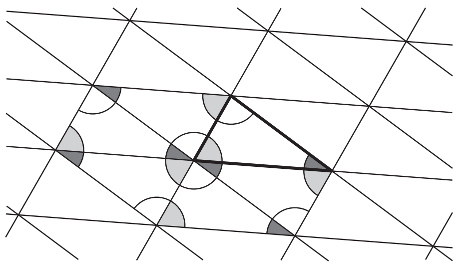
If you walk all the way around the edge of the triangle, you end up facing the way you began. You must have turned a total of 360˚.

You can see that each exterior angle when added to the interior angle must give 180˚ because they make a straight line.
This makes a total of 540˚.
$540˚ - 360˚ = 180˚$.

So Erin says it's true

Frank's answer

I drew a tessellation of triangles and marked all the equal angles.



I know that the angles round a point add up to 360˚.

So Frank says it's true

a) Whose answer do you like best? . . . .

b) Whose answer is closest to what you would do? . . . .

c) Whose answer would get the best mark from your teacher? . . . .

10 *Continued*

Please leave blank

d) For each of the following, circle whether you agree, don't know, or disagree.

The statement is:

When you add the interior angles of any triangle, your answer is always 180˚.

	agree	don't know	disagree
Asim's answer ... shows you that the statement is **always true**	1	2	3
Beth's answer ... shows you that the statement is **always true**	1	2	3
Cara's answer ... shows you that the statement is **always true**	1	2	3
Declan's answer ... shows you that the statement is **always true**	1	2	3
Erin's answer ... shows you that the statement is **always true**	1	2	3
Frank's answer ... shows you that the statement is **always true**	1	2	3

HG2

e) Suppose it has now been proved that:

When you add the interior angles of any triangle, your answer is always 180˚.

Zak asks what needs to be done to prove whether:

When you add the interior angles of any right-angled triangle, your answer is always 180˚.

Tick (✓) either A or B.

(A) Zak doesn't need to do anything, the first statement has already proved this. ❑

(B) Zak needs to construct a new proof. ❑

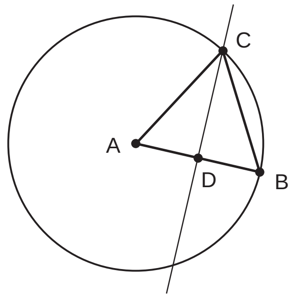
11 Prove whether the following statement is true or false. Write your answer in a way that would get you as good a mark as possible.

If you add the interior angles of any quadrilateral, your answer is always 360˚.

Please leave blank

HG4

12 A is the centre of a circle and AB is a radius.
C is a point on the circumference where the perpendicular bisector of AB crosses the circle.
Prove whether the following statement is true or false. Write your answer in a way that would get you as good a mark as possible.



Triangle ABC is always equilateral.

Please leave blank

HG7

WAIT! *Please go back to any questions you left out, then check all your answers.*
After that, if there is any time left over, please answer this questionnaire:

Please leave blank

Z1 a) What did you feel about taking part in this survey?

b) Which question did you like best, and why?

c) Which question did you like least, and why?

d) Please add any other comments, if you wish to, about the survey.

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Appendix B   Y10 Teacher Questionnaire

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Teacher Questionnaire (Y10)

Name ....

School ....   LEA   ....

Name of your Y10 maths class with students involved in the survey   ....

Please complete this questionnaire while your students are taking the proof survey.

Complete the details above and on pages 1 and 2, then work through the proof questions that follow.

Sch

Cla

Tea

Longitudinal Proof Project

Funded by the Economic and Social Research Council

Please tick (✓) the appropriate boxes and complete the appropriate blanks

Female ❑ 1 Male ❑ 2

How many years teaching experience did you have at the start of this school year? ....

Your age: under 25 ❑ 1 25 - 29 ❑ 2 30 - 39 ❑ 3 40 - 49 ❑ 4 50 - 59 ❑ 5 60 or more ❑ 6

School responsibility:

Head of maths ❑ Other (please specify) ....

Teaching Qualification:

Please specify type of qualification and subjects studied

Type		Main subject (please specify)	Subsidiary subject (please specify)
Degree (if not BEd)	❑	....	....
BEd	❑	....	....
PGCE	❑	....	....
Cert Ed	❑	....	....
Other	❑	....	....

Higher Education (apart from above):

Please specify type of qualification and main subject studied

Type		Title (eg MEd)	Main subject	Year completed
Masters	❑	....	....	....
PhD	❑	....	....	....

1

Continuing Professional Development (CPD) or INSET in mathematics education

a. In this section do NOT include Government INSET for NNS or NOF, or any courses that you have mentioned in the Higher Education section, but DO include activities such as attending courses or conferences, writing text books, serving as an examiner, taking part in projects.

For the previous school year (2000 - 2001), estimate the number of *sessions* you were involved in CDP or INSET in mathematics education (where a session is a morning, afternoon, twilight or evening): ....

b. Current membership of a professional association:

ATM ❑ MA ❑ IMA ❑ Other (please specify) ....

Involvement in extra-curricular mathematics activities with students in your school during 2001 - 2002 (ie activities that are not part of the normal school mathematics curriculum, such as organising a mathematics club, organising students for master classes or UK Maths Challenge, taking students to mathematics events):

Yes ❑ 1 No ❑ 2

Software that you have used this school year with Y10 students:

Logo ❑ Dynamic geometry ❑ Spreadsheet ❑ Database ❑

Graphics calculator ❑ Integrated learning system ❑ SMILE software ❑ The internet ❑

Other (please specify) ....

2

A3 Aysha, Brian, Coby, Deon, Eric and Fiona were trying to prove whether the following statement is true or false:

When you add any 2 even numbers, your answer is always even.

a) Please take a minute to think how you would prove this, then go on to part b).

b) Consider Aysha, Brian, Coby, Deon, Eric and Fiona's answers on the next page.

i. Give a mark (out of 10) for each answer. A . . . . B . . . . C . . . . D . . . . E . . . . F . . . .

ii. Whose answer would your students say would get the best mark from you? . . . .

iii. Whose answer is closest to what you would do? . . . .

c) Write a brief comment that might help these two students to move on:

Brian . . . .

. . . .

. . . .

. . . .

Coby . . . .

. . . .

. . . .

. . . .

Aysha's answer

a is any whole number.

b is any whole number.

$2a$ and $2b$ are any two even numbers.

$2a + 2b = 2(a + b)$.

So Aysha says it's true

Brian's answer

$2 + 2 = 4$	$4 + 2 = 6$
$2 + 4 = 6$	$4 + 4 = 8$
$2 + 6 = 8$	$4 + 6 = 10$

So Brian says it's true

Coby's answer

Even numbers are numbers that can be divided by 2. When you add numbers with a common factor, 2 in this case, the answer will have the same common factor.

So Coby says it's true

Deon's answer

Even numbers end in 0, 2, 4, 6 or 8. When you add any two of these the answer will still end in 0, 2, 4, 6 or 8.

So Deon says it's true

Eric's answer

Let x = any whole number, y = any whole number.

$x + y = z$

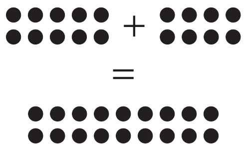
$z - x = y$

$z - y = x$

$z + z - (x + y) = x + y = 2z$

So Eric says it's true

Fiona's answer



So Fiona says it's true

G3 Asim, Beth, Cara, Declan, Erin and Frank were trying to prove whether the following statement is true or false:

When you add the interior angles of any triangle, your answer is always 180˚.

a) Please take a minute to think how you would prove this, then go on to part b).

b) Consider Asim, Beth, Cara, Declan, Erin and Frank's answers on the next page.

i. Give a mark (out of 10) for each answer. A . . . . B . . . . C . . . . D . . . . E . . . . F . . . .

ii. Whose answer would your students say would get the best mark from you? . . . .

iii. Whose answer is closest to what you would do? . . . .

c) Write a brief comment that might help these two students to move on:

Asim . . . .

. . . .

. . . .

. . . .

Declan . . . .

. . . .

. . . .

. . . .

Asim's answer

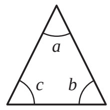
I tore the angles up and put them together.

It came to a straight line which is 180˚.
I tried for an equilateral and an isosceles as well and the same thing happened.

So Asim says it's true

Beth's answer

I drew an isosceles triangle, with c equal to 65˚.

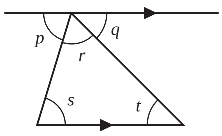


Statements	*Reasons*
$a = 180^\circ - 2c$	Base angles in isosceles triangle equal
$a = 50^\circ$	$180^\circ - 130^\circ$
$b = 65^\circ$	$180^\circ - (a + c)$
$c = b$	Base angles in isosceles triangle equal
$\therefore\ a + b + c = 180^\circ$.	

So Beth says it's true

Cara's answer

I drew a line parallel to the base of the triangle.



Statements	*Reasons*
$p = s$	Alternate angles between two parallel lines are equal
$q = t$	Alternate angles between two parallel lines are equal
$p + q + r = 180^\circ$...	Angles on a straight line
$\therefore\ s + t + r = 180^\circ$.	

So Cara says it's true

Declan's answer

I measured the angles of all sorts of triangles accurately and made a table.

They all added up to 180˚.

a	*b*	*c*	*total*
110	34	36	180
95	43	42	180
35	72	73	180
10	27	143	180

So Declan says it's true

Erin's answer

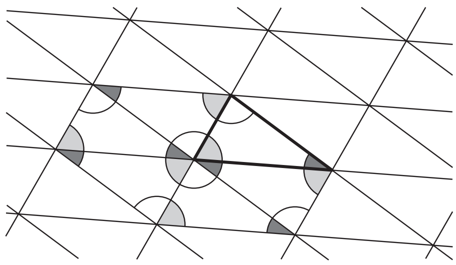
If you walk all the way around the edge of the triangle, you end up facing the way you began. You must have turned a total of 360˚.

You can see that each exterior angle when added to the interior angle must give 180˚ because they make a straight line. This makes a total of 540˚. $540^\circ - 360^\circ = 180^\circ$.

So Erin says it's true

Frank's answer

I drew a tessellation of triangles and marked all the equal angles.



I know that the angles round a point add up to 360˚.

So Frank says it's true

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Appendix C   Y10 School Questionnaire

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School Questionnaire (Y10)

Name of person completing questionnaire ....

School ....   LEA ....

Please complete this questionnaire at a convenient time and keep it with the other completed project materials.

Complete the details above and overleaf.

Sch

Longitudinal Proof Project

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School data

Please tick the boxes which best describe your school

Selection		Single-sex/mixed		Area	
No academic selection	☐ 1	Girls-only	☐ 1	Urban	☐ 1
Some academic selection	☐ 2	Boys-only	☐ 2	Rural	☐ 2
Full academic selection	☐ 3	Mixed-sex	☐ 3	Suburban	☐ 3

Year 10 data

Approximate number of Y10 students in the school ………………

How are the current Y10 classes organised? *Please tick one box*

Set ☐ 1 Banded ☐ 2 Mixed ability ☐ 3 Other ☐ 4

For the Y10 students taking the Proof Survey, please give the approximate percentage of students from their class who you predict will be entered for the GCSE Higher tier ………………

(If the students come from more than one class, please give the class name and percentage for each class)

Mathematics curriculum data

GCSE examination syllabus ………………………………………………………………………………

Main textbook / scheme in Year 10 ………………………………………………………………………………

Total duration (in minutes) of Y10 mathematics lessons per week ………………………………………………………………………………

Is the department currently involved in the CAME project? Yes ☐ 1 No ☐ 2

Extra-curricular mathematics activities

Are any Y10 students involved (at school or elsewhere) in any mathematics activities that are not part of the normal school mathematics curriculum (eg maths club, master classes, UK Maths Challenge)?

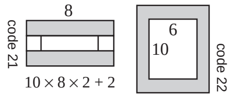
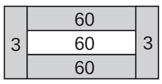
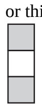
Yes ☐ 1 No ☐ 2

If YES, please describe:

………

Appendix D Y10 Coding Scheme and Scores

Coding Sheet Year 10 Proof Survey (and scores) combined version June 2003

Qu	Description of response and key points	Score
A1 11 12 13 14	*Generating data, spotting patterns, no structure* Answer = 180 + no reason or unclear + incorrect scalar strategy (10 × 18) + uses incorrect functional str (3 × 60) + incorrect scalar and functional Incorrect SCALAR (12): 6 → (× 10) → 60; 18 → 180 Incorrect FUNCT'L (13): 6 → 18; (× 3) 60 → 180	0
21 22	*Some recognition of structure but incomplete or goes wrong or no reason or draws and counts* Answer = blank/wrong: partial structure (eg doubles but does not add 6), or generates correct data (eg, 6,18 7,20, 8,22) but stops or goes wrong Answer = wrong: generates correct data (eg 10,26) but scales up (eg 10,26 × 6 gives 60,156) Answer = wrong: sees correct structure initially but does not apply correctly, eg scales up inappropriately (eg 2 lots of 10×8, plus 1+1). Answer = 36: sees white tiles as 10 rows of 6.  code 21: 8; 10 × 8 × 2 + 2 — code 22: 6, 10	1
23 23T 24	Answer = 126 + no reason or no clear reason or possibly false reason Answer = 126 + erroneous use of (erroneous) table (eg sees white tiles in rows of 6, gets 6,18 and 16,20 and uses +2 → ×2 to get g=2w+6). Answer = 126 (or close to 126): eg, draws a **photo**-picture with 60 white tiles	2
30 30T	*Recognition and **use of** structure, specific* Answer = 126 Schematic diagram (not photo) or description of **60** white tiles surrounded by grey tiles. NOTHING MORE. As code 30 but includes a table (or list) of data.  (3, 60, 60, 60, 3) or **60** + **60** + 3 + 3, **60** × 2 + 6, 62 + 62 + 2 white/grey: 1 8; 2 10; 3 12; 4 14 — + table = 30T	3
41 41T	*Recognition and **use of** structure, general, SCALAR* Answer = 126: sees that there are 10 times as many white tiles as in the given diagram, so will need 10 times as many grey tiles as there are above and below the given white tiles, plus the 3 tiles at each end: 10 × 12 + 6. **No explicit naming of variables.** Answer = 126: makes a table, using the idea that for every 6 extra white tiles that are 12 extra grey tiles. May go all the way to 60, 126. 6 → (× 10) → 60; 12 + 6 → 120 + 6 41T table — white/grey: 6 18; 12 30; 18 42; 24 56	3
42 42T	*Recognition and **use of** structure, general, FUNCTIONAL* Answer = 126 Shows procedure for getting the number of grey from the number of white: eg, always two lots plus 6; double and add 6; 2 greys for each white; × 2 + 6; times 2 add 6. **No explicit naming of variables** (so eg does **not** use "white" to refer to *total* whites). Less emphasis on 60: concentrates on ops of × and +. As code 42 but draws a table (or list) of data. might have similar drawing to code 30, or this:  white/grey: 1 8; 2 10; 3 12; 4 14 — + table = 42T	3
50 50T 50L 50LT	*Towards Algebra: naming variables* Answer = 126: as code 42 but also names one or both **variables** in **words** (and may expresses general relationship between variables): eg, The *number of grey* is 2 times the *number of white* plus 6, or Double the *amount of white tiles*, add 6, or Double the *white tiles*, add 6, or *white* × 2 add 6. As code 50, but uses a table (as in 30T, 41T, 42T). As code 50, but includes use of **letters**, eg 2*w* + 6, or 2*w* add 6. As code 50L, but uses a table.	3 3
91	*No response* 92 *No time (or informative non response)* 93 *Miscellaneous*	0

NOTE: Do not penalise *purely* arithmetical errors (eg 2×60 = 100) but add E to the code
Do not penalise *purely* counting errors (eg code 22, 24) but add E to the code

1

Qu	Description of response and key points - BRIEF	Score
A1b	*Passive description or pattern spotting*	
11	<u>Answer</u> = 6n or 60n (or 6n=18g, 60n=126g, etc). Add W, Y.	
12	<u>Answer</u> = 3n. Add W, G, Y.	
13	<u>Answer</u> = 10n. Add W, G, Y.	0
	Partial structure, or correct structure inadequately expressed (no letter or letter as object)	
21	<u>Answer</u> = 2n. Add W, G, Y.	
22	<u>Answer</u> = 2n+3. Add W, G, Y.	
23	<u>Answer</u> = ×2+6 Add G (for 2g + 6, 2n + 6g)	1
	Correct structure, correctly expressed	
30	<u>Answer</u> = 2n + 6	
30W	<u>Answer</u> = 2w + 6	
30Y	<u>Answer</u> = 2white + 6	2
91	*No response*	
92	*No time or informative response*	
93	*Miscellaneous*	0

Add W for w,
G for g (**except for code 30**),
Y for White. Add P for Power (in particular for $n^2 + 6$). Add B for correct 'back to front' expression.
Treat all letters other than w and g the same as n. Treat "grey" the same as "g".

Qu	Description of response and key points - LONG	Score
A1b	*Passive description or pattern spotting*	
11	<u>Answer</u> = 6n or 60n (or 6n=18g, 60n=126g, etc).	0
11W	<u>Answer</u> = 6w or 60w, etc.	
11Y	<u>Answer</u> = 6 white or 6 × white, etc.	
12	<u>Answer</u> = 3n.	
12W	<u>Answer</u> = 3w.	
12G	<u>Answer</u> = 3g.	
12Y	<u>Answer</u> = 3 white, or 3 × white, etc.	
13	<u>Answer</u> = 10n.	
13W	<u>Answer</u> = 10w.	
13G	<u>Answer</u> = 10g.	
13Y	<u>Answer</u> = 10 white, or 10 × white, etc.	
	Partial structure, or correct structure inadequately expressed (no letter or letter as object)	
21	<u>Answer</u> = 2n	1
21W	<u>Answer</u> = 2w.	
21G	<u>Answer</u> = 2g.	
21Y	<u>Answer</u> = 2 white or 2 × white, etc.	
22	<u>Answer</u> = 2n+3.	
22W	<u>Answer</u> = 2w+3.	
22G	<u>Answer</u> = 2g+3 or 2n + 3g.	
22Y	<u>Answer</u> = 2 × white + 3.	
23	<u>Answer</u> = ×2+6.	
23G	<u>Answer</u> = 2g + 6, 2n + 6g.	
	Note: It is possible to have • code 22WG (2w+3g) and 22GY (2white+3g) • code 23WG (2w + 6g) and 23GY (2white + 6g)	
	Correct structure, correctly expressed	
30	<u>Answer</u> = 2n + 6	2
30W	<u>Answer</u> = 2w + 6	
30Y	<u>Answer</u> = 2white + 6	

Note: Codes 91, 92, 93 not listed for this question, but use them in the usual way

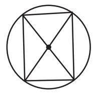
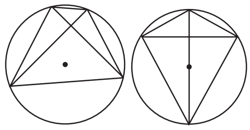
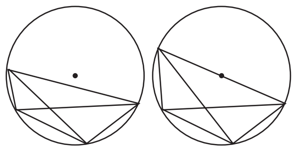
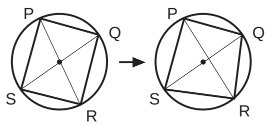
LA1 (a)	Yes	10	0		**LA1 (b)**	✓	sum is EVEN	30	2
	Yes changed to No	31	1			✓	sum is ODD	93	0
	No	32	2			✓	can't be sure	10	0
						✓	more than one	93	0

L1c	*(Correct or incorrect) decision: no valid justification*	
11	Yes + nothing, or unclear* or vague (eg "If you test it it makes sense"; "tried examples"; repeats) + examples where condition does not hold (eg, 3,4) + some examples that confirm and some examples that deny (eg, 3,5 and 4,6).	0
13	No + nothing, or almost nothing *(but confused = 93, here and in what would otherwise be code 11)	
14	No + example where **condition** does **not** hold, ie sum not even (eg, 2+5=7 and 2×5=10, so product not odd; or "even × odd is not odd" (ie could be specific or general example)	
	Decision with incomplete or flawed justification	
21	No + mixture of examples: condition does not hold (as in code 14) **and** valid counter example (as in code 31,32) (specific or general)	2
22	Yes + examples that **confirm only**, ie only odd numbers (eg, 3+5=8 and/or 3×5=15) (spec or gen).	
23	No + valid counter example (eg, 2,4) but incomplete (ie might consider sum but **not** product).	
	(Basically) correct decision + correct justification	
31	No + implicit counter example (eg, "2+4=6 and 2×4=8" or just "2×4=8") but does **not** say why example is important; may include non counter-examples that fit condition of sum=even, (eg, 1+3=4, 1×3=3) [Note: ignore examples which do not satisfy the condition (eg, 2,5 as in code 2) if it is clear that these are not meant to be part of the answer] (specific only)	2.5
32	No + explicit counter example (eg, [2+4=6 and] 2×4=8, *and 8 is **even***) ie states *why* the example is a counter example [Note: ignore examples which do not satisfy the condition (eg, 2,5 as in code 2) if it is clear that these are not meant to be part of the answer]	3
	Correct decision + general justification/description in narrative form	
4	No + If (A) the sum is even, then (B) the numbers could be both even, then (C) the product would be even B = code 41, A+B = code 42, B+C = code 43, A+B+C = code 44.	3
+L	As codes 41 to 44 + algebraic description of the set of counter examples (eg E+E=E, E×E=E).	
	Correct decision + general justification plus explanation of why justification is true	
50	No + as code 4, and adds explanation of **why** two evens have an even sum and/or product.	3
50L	As code 50 but uses algebra (eg, $2x + 2y = 2[x+y]$, $2x \times 2y = 4xy$). (use of E or O not sufficient here)	

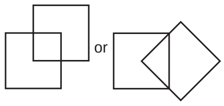
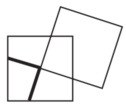
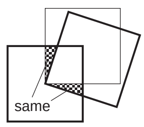
L1d	*(Correct or incorrect) decision: no valid justification*	
11	No + anything (including nothing)	0
13	Yes + nothing, or nothing sensible, or vague (eg "it always works"). + examples, some where condition does not hold, ie product not odd (eg, 2,4 or 3,4) (spec or gen) + repeat of statement (the product is odd, the sum is even) + false statement (specific or general)	
	Correct Decision + incomplete or limited justification, but not false	
21	Yes +confirmation by **one** empirical example	2
22	+confirmation by **several** empirical examples	
23	+confirmation by examples + recognition that this is not enough	
24	+crucial experiment i.e. random pair of odd numbers (eg, 19,23) [one or both numbers > 10].	
	Correct Decision + general justification of why numbers have to be odd + consequence:	
4	Yes + If (A) the product is odd, then (B) the numbers are both odd, then (C) the sum is even B = code 41, A+B = code 42, B+C = code 43, A+B+C = code 44.	3
+L	As codes 41 to 44 + algebraic description, eg "must be odd and O + O = E".	
	*Correct Decision + general justification plus explanation of **why** justification is true*	
50	Yes + As code 4 but adds explanation of **why** odd + odd is even (or why the numbers *have* to be odd)	3
50L	As code 50 but uses algebra (eg, $2x + 2y = 2[x+y]$, $2x \times 2y = 4xy$). (use of E or O not sufficient here)	

Parts c) and d):
try not to penalise pure arithmetic errors (but code as 93 if they lead to confusion);
also, code as 93 if question misunderstood [eg if they have not grasped the meaning of Sum and Product, or if they focus on the numbers that fit part b), or if they borrow from "You can't be sure ... until you know what the numbers are"];
code 12 abolished: For "same as Fred's/Joe's" or "same as below/above", code the answer in the other box as if it had been written in the box you are currently coding, and **add S for Same as Fred's/Joe's**.

3

G1	Description of response and key points	Score
11	*Incorrect decision: confirming example or no explanation* <u>Answer</u> = Yes + anything (including nothing) May have picture of quadrilateral where diagonals **do** cross at the centre 	0
12	*Correct decision but no explanation* <u>Answer</u> = No + nothing or not clear or not sensible. (Include answers that question the trustworthiness of the sketch but which say nothing about the quadrilateral itself, eg "Can't tell unless we use ruler and compass"[but this may be given a higher code if there is a diagram]).	1
21	*Correct but only implicit reasons: weak explanation* <u>Answer</u> = No + ambiguous or weak description of a counter example (or family of counter examples), including reference (without a diagram) to 'trapezium' or 'kite'. or + ambiguous or weak general explanation (<u>global</u> rather than analytic, ie concerned with the quadrilateral as a whole, rather than specifically with the endpoints of the diagonals), eg "quadrilaterals with different sides aren't symmetrical" "quadrilaterals don't all have right angles" "If the sides are much smaller on one side, the diagonals won't cross in the middle".	2
22	*Correct but only implicit reasons: weak diagram* <u>Answer</u> = No + ambiguous or weak diagram (ie the quadrilateral looks almost like a rectangle and the centre is near but not at the intersection of the diagonals, and there are no constraints on the quadrilateral, as opposed to 31). 	2
31	*Correct and explicit counter example* <u>Answer</u> = No + decisive <u>diagram</u> but decisiveness not 'absolute', (ie does not show that it would *never* be possible for the diagonals to meet at the centre with such a picture); <u>quadrilateral clearly not a rectangle</u>; possible reference to 'trapezium' or 'kite'; accept drawing with one diagonal going through centre, but only if not rectangle-like. 	2.5
32	<u>Answer</u> = No + clear description of counter example or absolutely decisive diagram(s) [but no dynamic argument (see code 41)]. Code 32: <u>Add D **or** N for Diagram **or** NoDiagram</u> 	3
41	*Correct analytic reason* <u>Answer</u> = No + use of dynamic argument, eg "One of the points (R) may be slightly offset, so the diagonal (PR) no longer goes through the centre". <u>Add D/N for Di/NoDi</u> 	3
42	<u>Answer</u> = No + clear general explanation (not dynamic=41 but nonetheless <u>analytic</u>, ie concerned with the endpoints of the diagonals rather than with the quadrilateral as a whole; but not simply a description of a counter example or family of counter examples), eg "The corners could be anywhere and the diagonals will not necessarily go through centre". <u>Add D/N for Di/NoDi</u>, <u>Add C for Cross</u> (ie diagonals only drawn)	3
91 92 93	*No response* *Informative no response* *Miscellaneous* (includes: illegible answers; diagram that does not satisfy conditions, eg draws arrowhead, where one vertex not on circle; Yes *and* NO; *neither* Yes *nor* No)	0

For codes 32, 41, 42: letter D **or** N **must** be added

G2b	*Specific estimate, close but wrong*	
11	Answer = 1/3 or 1/5 (or decimal equivalent) + any or no explanation.	0
12 13	*Correct decision but no structural explanation* Answer = $\frac{1}{4}$ + no explanation, or perception ("it looks like a quarter"), or spurious reason (the overlapping sides are halved and half times half is a quarter"). Add 'A' for 'always'. Answer = $\frac{1}{4}$ + actual, valid measuring (eg draws grid and counts, or measures right angled triangle and calculates). Add E for answers that are close to but not exactly 1/4. Add 'A' for 'always'.	1 1
20	*Correct decision but only implicit reasons* Answer = $\frac{1}{4}$ + sensible but only partial explanation (if obviously *not* sensible, then code 12). Could involve just one property ("corner is 90˚") but might involve several properties, and/or valid operations ("90˚ is a quarter of 360˚"; "You can divide the square into 4"); might include some reference to **turning** (but not as for code 31 or 32). Add 'A' for 'always'.	2
31 32	*Correct decision relating to case where obviously 1/4* Answer = $\frac{1}{4}$ + refers to turning square D so that it is oriented as in one of these diagrams or draws one of the diagrams (eg turn it 'to the side' or 'to the bottom' or 'till it is parallel'). Add 'A' for 'always', Add D/N for Di/NoDi  Answer = $\frac{1}{4}$ + claims that "the overlap fits 4 times", by referring to turning square D through successive 90˚ turns, or to partitioning the square into 4 equal parts, as in the diagram; or draws diagram. Add D/N for Di/NoDi 	3 3
40	*Explanation of 1/4 in general case* Answer = $\frac{1}{4}$ + uses 'compensation' argument to explain **why** rotating from simple case (code 31) conserves the area of overlap ("on one side it is covering slightly more of the square and on the other the same amount less"). Add D/N for Di/NoDi 	3
91	*No response* 92 *Informative no response* 93 *Miscellaneous wrong resp* (not 1/3, 1/4, 1/5)	0

Add 'A' for 'always' throughout

G4a		**Description of response and key points**	**Score**
ABC	A	*Calculating angle u*	
	4	Result of '360 – *p*' (normally "40"), somewhere on the page, with some evidence of where it came from.	1
	3	"40", somewhere on the page, but with no evidence of where it came from.	1
	2	A value for *u* other than 40, due to a *factual* error concerning 'angle at a point', eg "angle at a point = 380˚", or to a *method* error (eg "???"), or to an unknown error (but if clearly due just to an arithmetic error, give the appropriate code and add E at the end).	0
	0	No discernible value for *u* anywhere on the page.	
	B	*Calculating v + w*	
	4	Result of '180 – *u*' (normally "140"), somewhere on the page, with some valid evidence of where it came from.	0
	3	Result of '180 – *u*' (normally "140"), somewhere on the page, but with no evidence of where it came from.	
	2	A value for *v* + *w* other than 140, due to a *factual* error concerning 'angle sum of a triangle', eg "angle sum = 360˚", or to a *method* error (eg "???") or to an unknown error (but if clearly due just to an arithmetic error, give appropriate code and add E at the end).	
	0	No discernible value for *v* + *w*.	
	C	*Calculating v*	
	4	Result of '÷2' (normally "70"), somewhere on the page, with some valid evidence of where it came from.	0
	3	Result of '÷2' (normally "70"), somewhere on the page, but with no evidence of where it came from.	
	2	A value for *v* other than 70, due to a *factual* error concerning 'base angles of an isosceles triangle', eg "?????????", or to a *method* error (eg "*v* = *u*") or to an unknown error (but if clearly due just to an arithmetic error, give the appropriate code and add E at the end).	
	0	No discernible value for *v*.	
91		*No response at all*	
92		*Informative no response*	
93		*Miscellaneous* C and CE scores:	2

Where a *viable* parallel method is used, add P for Parallel method; try to fit to ABC coding, else code as 93P.
Add C for Correct to code if final answer is "70" (unless clearly obtained by wrong method).
Add CE if final answer is correct apart from arithmetic error.

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Version 01 15Dec02+

G4b	c=1/2n	Score
code 1	**Empirical**	
11	One example (even if wrong, as long as it is numerical)	0
12	More than one example	0
13	Crucial experiment (value of n not ending in 0 or 5 and greater than 10)	0
code 2	**[Exhaustive]**	
code 3	**[Enactive]**	
code 4	**[Naïve]** (use 50, 70)	
code 5	**Analytic formal**	
50	naive algebraic	0
51	1 correct algebraic expression (other than b=c)	0.5
52	2 correct algebraic expressions (other than b=c)	1
53	Correct derivation **Add M for Meet, S for Substitution**	2
code 6	**[don't use]**	
code 7	**Narrative**	
70	no structure (ie 'narrative-naïve')	0
71	Expresses one correct relationship	0.5
72	Expresses two correct relationships	1
73	Correct derivation **Add M for Meet, S for Substitution**	2
code 8	**[Visual]**	
code 9	Usual 91, 92, 93	0
~~99~~	~~counter example~~	

Version 01 15Dec02+

G4c	c=1/2n	
code 1	**Empirical**	
11	One example (c11,12,13: include pseudo example made to fit formula)	0
12	More than one example	0
13	Crucial experiment (value of n not ending in 0 or 5)	0
code 2	**[Exhaustive]**	
code 3	**[Enactive]**	
code 4	**[Naïve]** (use 50, 70)	
code 5	**Analytic formal**	
50	naive algebraic	0
51	1 correct algebraic expression (other than a=f)	0.5
52	2 correct algebraic expressions (other than a=f)	1
53	Correct derivation	2
code 6	**[don't use]**	
code 7	**Narrative**	
70	no structure (ie 'narrative-naïve')	0
71	Expresses one correct relationship	0.5
72	Expresses two correct relationships	1
73	Correct derivation	2
code 8	**[Visual]**	
code 9	Usual 91, 92, 93	0
~~99~~	~~counter example~~	

Codes 91, 92, 93 not listed for this question, but use them in the usual way

LG1 (a)	Yes Yes changed to No No	10 31 32	0 1 2		**LG1 (b)**	✓ ✓ ✓ ✓	both ACUTE not both ACUTE can't be sure more than one	30 93 10 93	2 0 0 0

LGc	*(Correct or incorrect) decision: no valid justification*	
11	Yes + nothing, or unclear* or vague (eg "If you test it it makes sense"; "tried examples"; repeats) + [examples where condition does not hold (eg, ??)] + some examples that confirm and some that deny (eg, 30˚,40˚,110˚ and 60˚,60˚,60˚).	0
13	No + nothing, or almost nothing (eg "If acute + acute, then acute"). *(but confused = 93, here and in what would otherwise be code 11)	
14	No + example where **condition** does **not** hold, ie does not start with two acute angles (eg "if 30˚ and 140˚, then third angle is 10˚ which is not obtuse") (could be specific or general example).	
	Decision with incomplete or flawed justification	
21	No + mixture of examples: condition does not hold (as in code 14) **and** valid counter example (as in code 31,32) (specific or general)	2
22	Yes + examples that **confirm only**, (eg, "30˚,30˚,120˚") (spec or gen) Add G for General (classic = "If acute + acute, then obtuse" = c22G, but also include part d) c4 type answer). Add B for Back to front deduction (eg "If 120˚, then 30˚+30˚" = c22B; "If obtuse, then acute + acute" = c22BG). Add V for Visual (eg a drawing of a confirming triangle, but with no specific values marked; or use of a visual argument, with or without diagram, eg "If you have a base with two acute angles, where they meet has to be obtuse"). Note: cannot have V *and* G.	
23	No + valid counter example (eg, "If 80˚+80˚") but incomplete (ie does not mention all the angles). Add N for "90˚-angled triangle" + nothing else (but give c32N for "90˚-angled triangle and two acute angles").	
	(Basically) correct decision + correct justification	
31	No + implicit counter example (eg, "80˚,80˚,20˚") but does **not** state that third angle is "acute" or "not obtuse"; may include non counter-examples that confirm (eg, "30˚,30˚,120˚"). [Note: ignore examples which do not satisfy the condition if it is clear that these are not meant to be part of the answer] (specific only).	2.5
32	No + explicit counter example (eg, "If 80˚+80˚ then 20˚, **and** 20˚ is acute/not obtuse") ie states *why* the example is a counter example. [Note: ignore examples which do not satisfy the condition if it is clear that these are not meant to be part of the answer.] (Add B for Back to front deduction) (can't see how this could appear here) Add M for Maximum (use of extreme values, 85˚-90˚ for acute angles, eg "89˚,89˚,2˚"). Add V for Visual (eg a drawing of a counter example, but don't add V if specific values are marked on the diagram). Add T for equilateral Triangle (eg "could have 60˚,60˚,60˚") Add N for "No + 90˚-angled triangle has two acute angles", but only c23N for "No + right angled triangle").	3
	Correct decision + general justification/description in narrative form	
4	No + If (A) two angles are both <90˚, and (B) their sum ≥ 90˚, then (C) the third angle ≤ 90˚ B = code 41, A+B = code 42, B+C = code 43, A+B+C = code 44. (Only give c44 if there is a clear awareness that the sum of the two acute angles *can be* < 90˚.) (Give c32N for "No + right angled triangle has two acute angles", but only c23N for "No + right angled triangle").	3
+L	As codes 41 to 44 + algebraic description of the set of counter examples (eg if X+Y≥90, then Z<90). Add B, V (although B unlikely, and V more likely under c2 and c3 than here; don't assume that a simple drawing with unspecified angles is 'general'). c4 answers involve a *range* of values, but this need be explicit in only one of the statements (A), (B), (C).	
	Correct decision + general justification plus explanation of why justification is true	
50 50L	Code 5 redundant here. DO NOT USE. Covered by c4R.	

LGd 11 13	*(Correct or incorrect) decision: no valid justification* No + anything (including nothing) (unless c93, see below) Yes + nothing, or nothing sensible, or vague (eg "it always works"). + examples, some where condition not hold, ie first angle not obtuse (eg, right triangle) (spec or gen) + repeat of statement ("If 1 obtuse, then 2 acute"), or repeat of statement back to front + false statement (specific or general) (unless c93, see below)	0
 21 22 23 24	*Correct Decision + incomplete or limited justification, but not false* Yes + confirmation by **one** empirical example + confirmation by **several** empirical examples + confirmation by examples + recognition that this is not enough + crucial experiment i.e. 'random' obtuse angle (eg 147˚) [not ending in 0 or 5, not 90-95˚, not 113˚]. Add B Add V Add M. [Use M for Minimum values 90˚ - 95˚. However, if a *range* is implied (eg "smallest value is 91˚"), then it might fit c4 (but this does not apply in part c).] [If the acute angles are left indeterminate, still use c2 if a specific value is given to the obtuse angle (eg, "If the obtuse angle is 130˚, then the other two add up to only 50˚"]	 2
 31 32	*Correct Decision + counter-counter example (proof by contradiction)* Yes +argument that there can't be another obtuse angle (as angle sum would be > 180˚) (but "so other angles must be acute" left implicit). Yes +argument that there can't be another obtuse angle (as angle sum would be > 180˚) (and "so other angles must be acute" made explicit). Add V (eg, add V for "Can't have two obtuse angles as lines would not meet to make 3rd angle").	 2.5 3
 4 +L	*Correct Decision + general justification of why resulting angles have to be acute:* Yes + If (A) one angle > 90˚, then (B) the sum of the other two ≤ 90, so (C) they are both acute B = code 41, A+BorC = code 42, B+C = code 43, A+B+C = code 44. (Give c13 or c93 for 'Yes+any mention of "right angled triangle"'). As codes 41 to 44 + algebraic description, eg " eg if Z>90, then X+Y≤90". Add B (though B unlikely here) Add V (though V unlikely here) ~~Add M~~ (redundant here; only allow a minimal value in c4 if it is *stated* that it is a minimal value) c4 answers involve a *range* of values, but this need be explicit in only one of the statements (A), (B), (C).	 3
 50 50L	*Correct Decision + general justification plus explanation of **why** justification is true* Code 5 redundant here. DO NOT USE. Covered by c4R.	

Parts c) and d):
try not to penalise pure arithmetic errors (but code as 93 if they lead to confusion);
also, code as 93 if question misunderstood [eg if they have not grasped the meaning of Acute and Obtuse, or if they think the angle sum is 360˚, say, or if they focus on values that fit part b) (113˚), or if they borrow from "You can't be sure ... until you know what the angles are"];
code 12 abolished: For "same as Rose's/ Kath's" or "same as below/above", code the answer in the other box as if it had been written in the box you are currently coding, and add S (for L1c) or S (for L1d) to the code.

HA4	Description of ODDS	Adding Odds
code 1	**Empirical**	
11	One example	0 = no calculation (is this likely) 1 = just does calculation(s) 2 = '1' and states the result is even **Add E for Exhaustive (or systematic)** **Add T for 2-wayTable format**
12	More than one example	
13	Crucial experiment (at least one number > 10 **and** 'random')	
code 2	**Exhaustive**	
21	Odd numbers end in 1,3,5,7,9	+0 = nothing new +1 = "result of adding end digits is even", or "result is 0,2,4,6,8" or list of actual results +1 = reference to the result being the 'end digit(s)' +1 = Line 3 + "and so the answer is even" (So to get a '3' here, need to be *close* to saying "the end digit will be even, which means the answer will be even") **Add T for 2-wayTable format**
22	moderately successful attempt at exhaustive list of end-digit calculations (eg incomplete but systematic)	
23	impressive attempt at exhaustive list of end-digit calculations	
code 3	**[Enactive]**	
code 4	**[Naïve]**	
code 5	**Analytic formal (correct)**	
51	2a+1	1 = write an expression for the sum, or just for the end bit, ie 1+1=2 2 = produces expression of the form 2 × A, where A is an expression, or perhaps just a single letter OR a statement that expression consists of a sum of several EVEN numbers 3 = Line 2 + "so the answer is even" (So to get a '3' need to be *close* to saying "2 × A is even" OR close to including "Even+Even is Even") **Add A for pureAlgebra**
52	2a+1, 2b+1	
code 6	**Analytic semi-formal**	
60	use of letter(s) but no structure (eg a = odd)	1 = write an expression for the sum, or just for the end bit, ie 1+1=2 2 = produces expression of the form 2 x A, where A is an expression, or perhaps just a single letter OR a statement that expression consists of a sum of EVEN numbers 3 = Line 2 + "so the answer is even" (So to get a '3' need to be *close* to including "Even+Even is Even") **Add A for pureAlgebra**
61	a is even, a+1 is odd	
62	a and b are even, a+1, b+1 are odd	
code 7	**(Analytic) narrative**	
70	no structure (ie 'narrative-naïve')	0 = nothing more 1 = extra 1 + extra 1 = 2 or even 2 = so the result consists of a sum of several EVEN numbers 3 = '2' + and so the answer is even **Add G for <u>Generic</u> example**
71	Partial structure (eg general idea that odds go up in 2s)	
72	ODD is EVEN+1 (general explanation)	
code 8	**Visual**	
80	no structure shown for individual odd numbers	1 = draws the result as a '2 by something' rectangle of dots +1 = indicates the structure of the result in some way (interlocking shapes, or the two 'odd' dots highlighted, or possibly referred to verbally) +1 = and so the answer is even **Add G for <u>General</u> 'example'** **Add V for pureVisual**
81	partial structure (eg number-line showing odds go up in 2s)	
82	Oddness Structure shown (generic example)	
code 9	Usual 91, 92, 93	
99	counter example	

Where students used **mixed styles** of answer, choose the dominant style or, if they can noy be separated out, the one that produces the **better proof**. Code the minor style (if there is one) in a separate column using just the first two digits of the usual 3-digit code.

<table>
<tr><td>HA7</td><td>Description of (p+q) and (p−q)</td><td colspan="2">Multiplying (p+q) by (p−q)</td></tr>
<tr><td>code 1</td><td>Empirical</td><td colspan="2"></td></tr>
<tr><td>11</td><td>One example</td><td colspan="2" rowspan="3">0 = no calculation (is this likely)
1 = just does calculation(s)
2 = '1' and states "the result is an M4" (or "is true")
3 = demonstrates that the result is an M4
Add G for empirical Generalisation (eg "one bracket is always an M4")</td></tr>
<tr><td>12</td><td>More than one example</td></tr>
<tr><td>13</td><td>Crucial experiment (at least one number > 10 and 'random')</td></tr>
<tr><td>code 2</td><td>Exhaustive</td><td colspan="2"></td></tr>
<tr><td>21</td><td>-</td><td colspan="2" rowspan="3">This category will not apply. Students might produce an exhaustive argument to show, rather than just state, that (p+q) and (p−q) are even, but we are not interested in this 'history'.</td></tr>
<tr><td>22</td><td>-</td></tr>
<tr><td>23</td><td>-</td></tr>
<tr><td>code 3</td><td>[Enactive]</td><td colspan="2"></td></tr>
<tr><td>code 4</td><td>[Naïve]</td><td colspan="2"></td></tr>
<tr><td>code 5</td><td>Analytic formal (correct)</td><td colspan="2"></td></tr>
<tr><td>51</td><td>2a+1</td><td colspan="2" rowspan="2">1 = write an expression for the product
2 = produces expression of the form 4 × A, where A is an expression, or perhaps just a single letter OR a statement that "the expression consists of the product of several EVEN numbers (or the sum of several M4s)"
3 = Line 2 + "so the answer is an M4"
(So to get a '3' need to be close to saying "4 × A is an M4" OR close to saying "M2×M2 = M4")
Add A for pureAlgebra
Add M for Multiplies brackets (eg $p^2 - q^2$)</td></tr>
<tr><td>52</td><td>2a+1, 2b+1</td></tr>
<tr><td>code 6</td><td>Analytic semi-formal</td><td colspan="2"></td></tr>
<tr><td>60</td><td>use of letter(s) but no structure (eg a = odd)</td><td colspan="2" rowspan="3">1 = write an expression for the product
2 = produces expression of the form 4 × A, where A is an expression, or perhaps just a single letter OR a statement that "the expression consists of the product of several EVEN numbers (or the sum of several M4s)"
3 = Line 2 + "so the answer is an M4"
(So to get a '3' need to be close to saying "4 × A is an M4" OR close to saying "M2×M2 = M4")
Add A for pureAlgebra
Add M for Multiplies brackets (eg $p^2 - q^2$)</td></tr>
<tr><td>61</td><td>a is even, a+1 is odd</td></tr>
<tr><td>62</td><td>a and b are even, a+1, b+1 are odd</td></tr>
<tr><td>code 7</td><td>(Analytic) narrative</td><td></td><td>$p^2 - q^2$</td></tr>
<tr><td>70</td><td>no structure (ie 'narrative-naïve')</td><td rowspan="3">0 = nothing more
1 = E × E = E (and so is true)
2 = E × E = <u>M4</u>
3 = E is M2, so true
4 = M2 × M2 = M4
Add G for <u>Generic</u> example (?)</td><td rowspan="3">0 = nothing more
1 = Odd − Odd = Even

Add M for Multiplies brackets (eg $p^2 - q^2$)</td></tr>
<tr><td>71</td><td>Partial structure
(eg states that <u>one</u> of (p+q) or (p−q) is even; or that p=E+1)</td></tr>
<tr><td>72</td><td>(p+q) and (p−q) are <u>both</u> even
(or p^2=Odd, q^2=Odd)
(general explanation)</td></tr>
<tr><td>code 8</td><td>[Visual]</td><td colspan="2"></td></tr>
<tr><td>80</td><td>no structure shown for (p+q) and (p−q)</td><td colspan="2" rowspan="3">1 = draws the result as a '4 by something' rectangle of dots
+1 = indicates the structure of the result in some way (interlocking shapes, or the two 'odd' dots highlighted, or possibly referred to verbally) (CAN THIS HAPPEN??)
+1 = and so the answer is even (CAN THIS HAPPEN??)
Add G for <u>General</u> 'example' (CAN THIS HAPPEN??)
Add V for pureVisual (CAN THIS HAPPEN??)</td></tr>
<tr><td>81</td><td>partial structure (eg structure shown for one bracket)</td></tr>
<tr><td>82</td><td>Evenness Structure shown for (p+q) and for (p−q)
(generic example)</td></tr>
<tr><td>code 9</td><td>Usual 91, 92, 93</td><td colspan="2">Generally, code a mistake that leads to 'No' as c93.
Add D for Different values for each p and/or each q</td></tr>
<tr><td><s>99</s></td><td><s>counter example</s></td><td colspan="2"></td></tr>
</table>

Where students used **mixed styles** of answer, choose the dominant style or, if they can be separated out, the one that produces the **better proof**. Code the minor style (if there is one) in a separate column using just the first two digits of the usual 3-digit code.

11

HG4	Description of quadrilateral	Argument used
code 1	**Empirical**	
11	Rectangle or square (one or more)	0 = nothing, or bald statement that Σ=360˚ or "Measure and see..." 1 = 'demonstrates' that Σ=360˚ for chosen quadrilateral (eg a 90,90,90,90 square or a quad whose marked angles sum to 360). 2 = Expresses/uses an incorrect relationship (eg opposite angles of parallelogram=180˚) 3 = Expresses/uses a correct relationship (eg opposite angles of cyclic quadrilateral=180˚) but does not get to Σ=360˚ (eg next step incorrect) 4 = uses a correct relationship to get to Σ=360˚ for chosen quadrilateral. **Add X for Exterior angles, P for Parallel construction line(s), T for Triangles. N for NO diagram, D for Dynamic, B for Backwards (circular)**
12	Semi-generic (eg parallelogram, trapezium, kite)	
13	Generic (ie completely un-special)	
code 2	**[Exhaustive]**	
code 3	**Enactive**	Tears off corners
31	Rectangle or square (one or more)	0 = baldly states angles make 360˚ 1 = produces a drawing which shows four angle making a complete circle 2 = ? 3 = size of angles <u>or</u> labels (but not both) match angles in original quadrilateral 4 = size of angles <u>and</u> labels match angles in original quadrilateral
32	Semi-generic (eg parallel-ogram, trapezium, kite)	
33	Generic (ie completely un-special)	
code 4	**[Naïve]**	
code 5	**Analytic formal** (correct or not)	Uses a, b, c, d, say, for angles (in exposition, not just in diag)
50	no structure (ie 'analytic-naïve')	0 = nothing, or bald statement that Σ=360˚. 1 = 'demonstrates' that Σ=360˚ for chosen quadrilateral (not sure this is possible). 2 = Expresses/uses an incorrect relationship (eg opposite angles of parallelogram=180˚) or uses a false/irrelevant argument or rel 3 = Expresses/uses a correct relationship (eg opposite angles of cyclic quadrilateral=180˚) but does not get to Σ=360˚ (eg next step incorrect) 4 = uses a correct relationship to get to Σ=360˚ for chosen quadrilateral. **Add X for Exterior angles, P for Parallel construction line(s), T for Triangles (ie ΣT+ΣT=180+180=360)** **Add N for NO diagram, F for Formula (NO A for pureAlg)**
51	Rectangular or squarish shape (one or more)	
52	Semi-generic (eg parallelogram, trapezium, kite) (could include *some* specific angles, but must include indeterminate angles)	
53	Generic (ie completely un-special; completely general)	
code 6	**[Analytic semi-formal]** (use c5)	
code 7	**(Analytic) narrative**	Does <u>not</u> use a, b, c, d, say, for angles (in exposition, whether or not in diag)
70	no structure (ie 'narrative-naïve')	0 = nothing, or bald statement that Σ=360˚. 1 = 'demonstrates' that Σ=360˚ for chosen quadrilateral (is this possible?). 2 = Expresses/uses an incorrect relationship (eg opposite angles of parallelogram=180˚) 3 = Expresses/uses a correct relationship (eg opposite angles of cyclic quadrilateral=180˚) but does not get to Σ=360˚ (eg subsequent step might be incorrect) or uses a correct but not completely general rel to get to Σ=360˚ for not completely general quadrilateral 4 = uses a correct general relationship to get to Σ=360˚ for general quadrilateral (=top mark) **Add X for Exterior angles, P for Parallel construction line(s), T for Triangles (ie ΣT+ΣT=180+180=360) Add N for NO diagram Add D for Dynamic**
71	Rectangular or squarish shape (one or more)	
72	Semi-generic (eg parallelogram, trapezium, kite) (could include *some* specific angles, but must include indeterminate angles)	
73	Generic (ie completely un-special; completely general)	
code 8	**Visual**	Draws Tessellation (OR?)
80	[?]	0 = baldly states angles make 360˚ 1 = produces a drawing which shows four angle making a complete circle [2 = ?] 3 = size of angles <u>or</u> labels (but not both) match angles in original quad 4 = size of angles <u>and</u> labels match angles in original quadrilateral **(Add V for pureVisual)**
81	Rectangular or squarish shape	
82	Semi-generic (eg parallel-ogram, trapezium, kite)	
83	Generic (ie completely un-special; completely general)	
code 9	Usual 91, 92, 93	Generally, code a mistake that leads to 'No' as c93
~~99~~	~~counter example~~	

Where students used **mixed styles** of answer, choose the dominant style or, if they can be separated out, the one that produces the **better proof**. Code the minor style (if there is one) in a separate column using just the first two digits of the usual 3-digit code.

HG7	Description of triangle (Properties)	Argument used
code 1	**Empirical**	
11	Direct properties based on measurement	0 = nothing, or bald statement that Δ=Equilateral. 1 = 'demonstrates' that Δ=Equilateral by measuring 3 sides or 3 angles. (Note: Have not used this well or consistently: usually when they've drawn a confusing example have called it 700 rather than 110, say)
12	[-]	
13	[-]	
code 2	**[Exhaustive]**	
code 3	**[Enactive]**	
code 4	**[Naïve]**	
code 5	**Analytic formal (correct or not)**	Dominant style is 'AB=AC' rather than 'AB, AC same length'
51	some 'direct properties' (eg AB=AC, AD=DB, D=90, ΔADC=ΔBDC) but no reasons	0 = nothing, or bald statement that Δ=Equilateral. 1 = some 'derived' properties (CA=CB or AB=AC=CB, or equivalent) but no reason or explicit deduction 2 = "1" + some reason or deduction 3 = complete properties+deductions (ie CA=CB <u>and</u> AB=AC=CB, or equivalent). **Add S for use of Symmetry, C for use of Congruence, P for Pythagoras (NO A for pureAlg)**
52	some properties, some valid reasons (eg 'radii', 'bisector', 'perp')	
53	all properties needed for particular proof (eg AB=AC, AD=DB, D=90, <u>ΔADC=ΔBDC</u>), all with valid reasons	
code 6	**Analytic semi-formal (**Use c5 instead)	
code 7	**(Analytic) narrative**	May use some of 'AB=AC' but dominated by long thread of text
70	no structure (ie 'narrative-naïve')	0 = nothing, or bald statement that Δ=Equilateral (include explicit properties, if no reasons given) 1 = some 'derived' properties (CA=CB or AB=AC=CB, or equivalent) but no reason or explicit or valid deduction 2 = "1" + some valid reason or deduction 3 = complete properties+deductions (ie CA=CB <u>and</u> AB=AC=CB, or equivalent). **Add S for use of Symmetry, C for use of Congruence, P for Pythagoras**
71	some 'direct properties' (eg AB=AC, AD=DB, D=90, ΔADC=ΔBDC) but no reasons	
72	some properties, some valid reasons (eg 'radii', 'bisector', 'perp')	
73	all properties needed for particular proof (eg AB=AC, AD=DB, D=90, <u>ΔADC=ΔBDC</u>), all with valid reasons	
code 8	**(Visual)**	
code 9	Usual 91, 92, 93	Generally, code a mistake that leads to 'No' as c93
~~99~~	~~counter example~~	

Where students used **mixed styles** of answer, choose the dominant style or, if they can be separated out, the one that produces the **better proof**. Code the minor style (if there is one) in a separate column using just the first two digits of the usual 3-digit code.

Scores for HA4, HA7, HG4, HG7

Y10HA4	score
111	1
112	1
121	1
122	1
131	1
132	1
210	1
211	1
212	2
213	3
220	1
221	1
222	2
223	3
230	1
231	1
232	2
233	3
510	1
511	2
512	2
513	2
520	2
521	2
522	3
523	3
600	0
601	0
610	1
611	2
612	2
613	2
620	1
621	2
622	2
623	3
700	0
710	1
711	1
720	1
721	2
722	3
723	3
801	1
810	1
811	1
812	2
820	1
821	2
822	3
823	3
91	0
92	0
93	0

Y10HA7	score
110	1
111	1
112	1
113	1
120	1
121	1
122	1
123	1
131	1
132	1
133	1
511	1
512	2
513	2
521	1
522	2
523	3
601	0
610	1
611	1
612	2
613	2
700	0
701	0
710	1
711	1
712	1
713	2
714	3
720	1
721	1
722	2
723	2
724	3
821	1
91	0
92	0
93	0

Y10HG4	score
110	0
111	1
112	1
113	1
114	1
120	0
121	1
122	1
123	1
124	1
130	0
131	1
132	1
133	1
134	1
310	0
311	1
313	1
320	0
321	1
323	1
324	1
330	0
331	1
333	1
334	1
500	0
514	3
520	0
522	1
523	2
524	3
530	0
532	1
533	2
534	3
700	0
710	0
711	1
712	1
713	2
714	3
720	0
721	1
722	1
723	2
724	3
730	0
732	1
733	2
734	3
811	1
813	1
821	1
824	2
830	0
831	1
91	0
92	0
93	0

Y10HG7	score
110	0
111	1
500	0
510	1
521	2
700	0
701	1
702	2
710	1
711	1
712	2
720	2
721	2
722	2
730	2
731	2
732	2
733	3
91	0
92	0
93	0